Academic Senate of the California State University

Quantitative Reasoning Task Force

Final Report, August 1, 2016

Guiding Principle: Educational policy should balance access and opportunity to achieve equity.

Upon its acceptance by the Academic Senate of the California State University in September 2016, this report and its appendices will be posted under “Student Preparedness/Success” at http://www.calstate.edu/AcadSen/Records/Reports/index.shtml.

Executive Summary

In its 2015-16 term the Academic Senate of the California State University convened a Quantitative Reasoning Task Force to review the CSU's expectations for student proficiency in quantitative reasoning, both before college and at graduation, and to recommend changes to existing policies and practices. (See Appendix A, Academic Senate CSU Resolution 3230-15.)

The current CSU policies have set the standards for statewide curricula in quantitative reasoning for many years, raising concerns they lag current thinking and best practices in the field. Disturbingly, evidence also suggests that existing policies may present unreasonable barriers to student success, particularly for students from traditionally underserved populations, and especially in the California Community Colleges.

The Task Force included faculty and administration representing the CSU, the University of California, the California Community Colleges, the California Department of Education, employers, and the Office of the Lieutenant Governor. Its final recommendations were prepared by a subset of the Task Force holding offices in the Academic Senate CSU, and designated "drafting members." (See Appendix B, Task Force Membership.)

Members of the Task Force conducted an extensive literature review, met with invited advisors, and participated in a national forum programmed by the U.S. Department of Education and hosted at the CSU Office of the Chancellor.

The work of the Task Force was guided by the principle that **any educational policy enacted by the CSU must balance access and opportunity to achieve equity**. That is, genuine equity lies in providing students from all backgrounds with equitable prospects not only for admission and graduation, but also for meaningful degrees that prepare them for high-value careers and lives after graduation.

This report details the final recommendations of the Quantitative Reasoning Task Force, which are:

Recommendation I. Formulate an updated quantitative reasoning definition based on CSU best practices and reflecting national standards. Current policy relies on the phrase "intermediate algebra" as shorthand for full college preparation through high school, and defines baccalaureate-level quantitative reasoning as the math that builds on this level. The Task Force recommends updating this definition to include other kinds of quantitative reasoning.

Recommendation II. Revise CSU quantitative reasoning requirements, and adopt equitable, implementable requirements that articulate well with the other segments. The Task Force found that CSU policies with respect to admission, transfer, and graduation are unduly constrained by treating foundational quantitative reasoning as necessary for success in all kinds of baccalaureate-level quantitative reasoning. Better policies would recognize that quantitative reasoning is valuable at both levels in ways that aren't always sequential. The Task Force proposes flexible and appropriately rigorous definitions of

quantitative reasoning at the foundational and baccalaureate levels to inform separate requirements at entry and at graduation.

Recommendation III. Adopt implementable policies that ensure equitable access and opportunity to all CSU students. The Task Force recommends policy revisions to provide equitable treatment of community college transfer and native CSU students; improve access to quantitative reasoning classes relevant to a student's major, interests, and career; and raise the CSU systemwide expectation for quantitative reasoning in high school from three to four years.

In each case, the Task Force sought to make its recommendations equitable by balancing access and opportunity. For example, the recommendation to raise the CSU's systemwide expectation for quantitative reasoning coursework in high school to four years emphasizes that the fourth year of instruction should include practice and application of prior learning in quantitative reasoning, as opposed to a new course of topics in math. (In operational terms this means the fourth year of high school quantitative reasoning might not be in area "c" of the UC "a-g" curriculum of college preparatory courses.)

Recommendation IV. Create a CSU Center for Advancement of Instruction in Quantitative Reasoning. The Task Force appreciates the rapidly changing contexts of high school instruction, best practices in postsecondary education, and the skills in quantitative reasoning that CSU students will rely on after graduation. This report supports a recent resolution of the Academic Senate CSU calling for creation of such a Center, to act on these and subsequent findings and to continuously support the high-quality instruction and curriculum in the high schools, community colleges, and public universities that will better serve the state.

Although presented separately here, the four recommendations are interdependent. For example, the policy proposals in Recommendation III depend on the definitions and distinctions of Recommendations I and II. Especially crucial is the creation of Center (Recommendation IV), to be modeled on the CSU's successful Center for the Advancement of Reading. Members of the Task Force expressed reservations about reducing the emphasis on algebra unless rigor could be assured in other ways, with sustained system-level attention to pedagogy, evidence of learning at entry for both freshmen and transfer students, and support for high schools offering 12th grade courses in quantitative reasoning.

Introduction to CSU Quantitative Reasoning

Current Policies

Before Admission. As part of the Early Assessment Program, California 11th grade students take the California Assessment of Student Performance and Progress in English and mathematics, which provides an early indication of their readiness for college, while they still have time to schedule additional classes in the senior year if necessary. This Early Assessment Program is a collaborative effort among the California State University, the California Department of Education, and the State Board of Education. Currently the program uses the Smarter Balanced Summative Assessment in mathematics to measure student proficiency.

Upon Admission. Pursuant to Title 5 of the California Code of Regulations, the CSU requires that all admitted students "possess basic competence in … mathematical computation to a degree reasonably expected of entering college students." Further, the CSU must promptly identify students who "cannot meet such competence" and require they remediate any entry-level "deficiencies." To these ends, the CSU Chancellor issued Executive Order 665 (1997) to establish the Entry-Level Mathematics (ELM) examination and a committee for its maintenance. EO 665 Addendum A articulates entry-level expectations:

> *The ELM examination tests for entry level mathematics skills acquired through three years of rigorous college preparatory mathematics coursework (normally Algebra I, Algebra II, and Geometry).*

Addendum A also provides ELM test proxies (e.g., SAT, ACT, and Advanced Placement exam scores) for establishing basic competence. In the twenty years since the creation of the ELM test, there has been a decreased emphasis on second-year algebra and an increased focus on deeper mastery of the skills developed in Algebra I and Geometry, as evidenced in the list of Topics on the ELM test published at ETS.org. In 2002 developers revised the test to include more text-based and contextualized problems to assess quantitative reasoning in different situations and for different purposes. Of great concern to the Quantitative Reasoning Task Force is the fact that corresponding scores on the ELM test proxies such as the SAT weren't adjusted to match the new ELM test content.

Summer before Freshman Year. The Early Assessment Program has been nationally recognized for raising high school students' awareness of their readiness, and contributing to increased enrollment in 12th grade math and English. But in its first decade of implementation, rates of student readiness at entry remained flat, as documented by the proficiency reports at calstate.edu. In response the Trustees created the Early Start Program in 2010, subsequently codified in Executive Order 1048, which states:

> *incoming freshmen who have not demonstrated proficiency in English and/or mathematics will be required to begin remediation prior to the term for which they have been admitted, e.g., summer prior to fall.*

Implementation was phased in over several years, with the final phase completed summer of 2014. As of this writing a record share of the CSU's incoming freshmen are placed at college level, a success that the system attributes in part to the combined benefits of the Early Assessment Program and Early Start. From a March 2015 presentation to the Board of Trustees:

> *The Early Start program has successfully enhanced pre-existing campus and system efforts to improve the number of freshmen prepared for college-level mathematics and English when they begin their first term. In summer 2010, existing CSU programs improved proficiency in both English and mathematics by one percentage point resulting in 44 percent of the 2010 freshmen class starting their first term at the CSU college-ready in English and mathematics. Comparatively, summer 2014 Early Start courses improved proficiency in both English and mathematics by five percentage points resulting in 59 percent of the entering freshmen class being prepared for college-level English and mathematics.*

Prior to Graduation. As part of the General Education Breadth Requirements, Title 5 specifies that all graduating CSU students must complete at least 12 semester units (or 18 quarter units) that

> *… include inquiry into the physical universe and its life forms, with some immediate participation in laboratory activity, and into mathematical concepts and quantitative reasoning and their applications.* Title 5 §40405.1.

CSU Executive Order 1100 mandates that courses in subarea B4 (Quantitative Reasoning) of the GE Breadth curriculum

> *shall have an explicit intermediate algebra prerequisite, and students shall develop skills and understanding beyond the level of intermediate algebra. Students will not just practice computational skills, but will be able to explain and apply basic mathematical concepts and will be able to solve problems through quantitative reasoning.*

To comply with Executive Order 1100 and to qualify for the B4 designation, a course should include an intermediate algebra prerequisite. However, a review of system-wide approved B4 courses suggests that practices around CSU Area B4 graduation requirement – like the Entry Level Math examination – have evolved away from the reliance on intermediate algebra. The Task Force examined system-level data and used course titles to group courses and enrollments into four kinds of curriculum:

> **Algebra Not Calculus:** Courses that rely on some algebra concepts without explicitly preparing the student for eventual study of calculus. Business math is one example.
>
> **Calculus and/or Algebra:** Courses in traditional math sequences culminating in calculus or coming after calculus, and which are recommended preparation for the majority of STEM majors.

Statistics: Courses that emphasize statistical reasoning and don't necessarily prepare students for calculus. These are prevalent in some social science majors, and in some newer cases may not carry an explicit prerequisite of intermediate algebra.

Ideas in Quantitative Reasoning: Courses that emphasize quantitative reasoning for everyday life, and which are typically directed at non-STEM majors.

CSU campuses had an opportunity to correct these categorizations, and around a third offered minor adjustments. Table 1 displays the results.

Table 1: Quantitative Reasoning in the CSU - B4 Courses[1]

Fall 2013-2015	Number of Courses	Number of Enrollments
Algebra Not Calculus	17	18,963
Calculus and/or Algebra	111	143,012
Statistics	66	85,585
Ideas in Quantitative Reasoning	56	32,334

As Table 1 shows, from Fall 2013 to Fall 2015, the CSU campuses offered a total of 250 courses that satisfied the Area B4 quantitative reasoning requirement. Of these, 122—or nearly half—have titles such as statistics or "Ideas in Math" that suggest students will not be expected to use intermediate algebra. Approximately 42% of the students who enter the CSU as freshmen take these non-algebra-intensive courses to meet their GE requirements. (However, some CSU campuses require students taking such classes to pass an intermediate algebra test prior to enrolling, possibly to comply with the above mentioned executive orders.)

Issues of Inequity

Inequity in access for Developmental Math CSU First Time Freshmen. The intermediate algebra threshold does not reflect current CSU practice for entering freshmen. CSU freshmen may be deemed ready for B4 courses if they get a scaled score of 50 or better on the ELM exam. As the ELM exam tests for proficiency in Algebra I and some Geometry, but very little Algebra II (generally understood to be synonymous with intermediate algebra), students who enter the CSU as "proficient" as measured by the ELM exam are not necessarily proficient in intermediate algebra.

Those who enter the CSU as "not proficient" as measured by the ELM exam are required to complete developmental math work within their first year. This course work may or may not be held to the intermediate algebra standard (rather than the ELM exam standard) depending on which CSU campus the student attends. This can create a disparity between the standard for "proficient at entry" students and the standard for "not proficient at entry" students.

[1] See Appendix C, Course and Enrollment Data

Since EO 665 prescribes that "not proficient at entry" students must complete developmental math course work in a timely way or risk being "stopped out" from the CSU system, this disparity raises legitimate equity concerns.

Inequity in access for Transfer Students. In order to gain transfer admission to the CSU, community college students must provide evidence of satisfactory completion of an approved, quantitative reasoning course with an explicit intermediate algebra prerequisite. Community college students are placed into or out of college level math by a variety of placement tests depending on the campus. The goal of those placement tools is to determine whether students are proficient in intermediate algebra.

Thus, community college students are held to a stricter standard of math proficiency than are entering CSU freshmen. The Community College placement process results in as much as 85% of community college students taking sequences of developmental math courses. It is well documented that such sequences of courses, sometimes as long as three or four courses, result in very few students ever completing a college level math class. In fact, students who place into the lowest level of developmental math, have only a one in ten chance of ever doing so.[2] This raises a second equity concern.

Each year, the California Community Colleges (CCC) submit more than 1000 course outlines to the CSU for recognition in the GE Breadth and in the Intersegmental General Education Transfer Curriculum (IGETC) transfer patterns. Courses proposed for quantitative reasoning must demonstrate both an explicit intermediate algebra prerequisite and evidence that the course will build on algebra proficiency.

California's articulation records are stored in the ASSIST online database. A query of community college courses currently approved for transfer credit in Area B4 Quantitative Reasoning returned records for 1,616 separate courses. As it did with the B4 courses offered on CSU campuses, the Task Force grouped community college courses into four kinds of curriculum, and then invited the colleges to make any corrections. Around a quarter of the state's 113 community colleges replied, some with minor corrections and others to say the groupings were accurate as proposed.

The results in Table 2 indicate that transferable college level quantitative reasoning classes in the community college system are less varied than those in the CSU.

Table 2: Quantitative Reasoning in the CCC - B4 Courses[3]

	Number of Courses
Algebra Not Calculus	149
Calculus and/or Algebra	999
Statistics	272
Ideas in Quantitative Reasoning	196

[2] For an account of current placement policies see Burdman, *Degrees of Freedom*, 2015.

[3] See Appendix C, Course and Enrollment Data.

Around a quarter of the courses offered in community colleges are in statistics or ideas in quantitative reasoning, compared to around half in the CSU. Although this finding doesn't take community college enrollment into account, it suggests that community colleges apply CSU Executive Order 1100 more literally than do CSU campuses.

Since most graduates of the CSU initially enroll as transfer students, and since transfer students are a vital source of diversity and access to the baccalaureate, it follows that these differences in expectations and practices undermine the principle of equitable access to the CSU.

Inequity in opportunity for developmental math students. In response to the equity challenges above, a number of California Community Colleges and a few CSU campuses have been piloting statistics pathways for students in non-math intensive majors. Under temporary approvals from the CSU General Education Advisory Committee (GEAC), these pathways counted for lower division CSU quantitative reasoning credit. At its meeting of September 2015, GEAC heard reports of improved passage rates for students in the statistics pathways, both in GE quantitative reasoning courses and in some cases in subsequent lower division GE coursework that relies on quantitative reasoning. These pathways also significantly narrowed or closed racial equity gaps in completion of baccalaureate level quantitative reasoning courses. Such studies suggest that a pathways approach is a potential solution to the inequities of access mentioned above.

However, GEAC and others have raised concerns about the effect of such pathways on the other side of equity: opportunity. The absence of specific algebra requirements in these pathway programs raised concerns on the part of the CSU Council of Math Chairs and GEAC about a possible erosion of the value of a CSU bachelor's degree. The promising early evidence of success was considered noteworthy but on its own not definitive, prompting the creation of this Task Force. Worries about the erosion of the degree tended take two forms:

At a general level, CSU faculty expressed flexibility about moving away from the intermediate algebra threshold but wished to do so in a way that ensured that future students are prepared to apply quantitative reasoning skills as educated and responsible lifelong learners, in fields like personal finance (e.g. compound interest rates) or the topics found in general education classes like environmental science or geology, or the Science, Technology, Engineering, and Math (STEM) courses taken by a broad range of majors.

A second, more specific concern was that a revised threshold could result in channeling students from underserved communities into less lucrative and secure careers. This concern comes from the intentional design of statistics pathways. Statistics pathways are designed for students placed into remediation who plan to major in non-algebra intensive fields. A statistics pathway is not appropriate for students in STEM or business programs. Thus, a statistics pathway does not prepare students for careers in these fields and most Task Force members were comfortable with this level of tracking students.

However, there was additional concern that the level of quantitative reasoning preparation in the temporarily approved statistics pathways curricula could leave students unprepared even for non-algebra-intensive careers that require some algebra proficiency. For example,

nursing programs that require physics would need more algebra than a statistics pathway would provide. The Task Force heard concerns from experts in math education about the appropriateness of statistics pathways for elementary school teachers. Since teaching and nursing are two common careers that provide an entrée to the middle class, many Task Force members felt that these concerns should be weighed against the opportunity that statistics pathways offer for access to a baccalaureate degree for students in other programs.

All agreed that if students have a choice of math pathways, they must be informed up front with career exploration opportunities, proper advising, curricular maps, and meta-majors groupings to ensure their choices reflect their own aspirations and dreams and not their fears of mathematics.

Without reaching agreement on the merit of these specific concerns, supporting arguments, and counter-arguments, the Task Force acknowledged the importance of analyzing equity implications of its decisions and supporting the premise that genuine equity demands both access to and the opportunity afforded by the degree.

Inequitable outcomes in baccalaureate-level quantitative reasoning courses in the CSU. The CSU Office of the Chancellor provided the Task Force with detailed enrollment data from the Fall 2013 term through Fall 2015, including pass rates for each of the courses tabulated in Table 1. Student outcomes were disaggregated by ethnic and racial groups following national practice: African-American, Latino, and American Indian students are grouped together as so-called "Under-Represented Minority" populations, while all other students are grouped separately, sometimes called non-URM, as a way of identifying inequitable outcomes.

The findings are consistent with national research, indicating greater differences in pass rates for algebra-intensive courses in quantitative reasoning.

Table 3: CSU Student Outcomes in B4 Courses, F13 through F15[4]

	Pass Rates for Latinos, African-Americans, American Indians	Pass Rates for Other Populations	Difference in Pass Rates
Algebra Not Calculus	70.77%	81.27%	10.50 pts
Calculus and/or Algebra	67.21%	76.89%	9.67 pts
Statistics	75.26%	84.74%	9.48 pts
Ideas in Quantitative Reasoning	79.94%	87.13%	7.20 pts

Goal of the Quantitative Reasoning Task Force. The Task Force sought to examine and address the asserted inequities in both access and opportunity, while creating a more current and transparent set of published criteria within which all public education segments can innovate.

To attend to equity issues related to *opportunity*, the Task Force took the view that quantitative reasoning is more than just one course to satisfy a general education

[4] See Appendix C, Course and Enrollment Data.

requirement. It is the quantitative work necessary to support a student's major, interests, career and civic responsibilities.

Out of concern for equity issues related to *access*, the Task Force was careful that any proposed standards are justified by demonstrably valuable learning. It also attended to the need for any evolving standard to integrate well with the curricula of our sister institutions. In doing so, the Task Force borrowed heavily from the high school segment as it formulated its recommendations with the California Common State Standards language. Similarly, the recommendations in this report were informed by innovations in quantitative reasoning education in community colleges in California and nationwide.

Crucially, the Task Force recommends shifting from relying on prerequisites from other institutions to invest quantitative reasoning value in the degree to relying on a sensible threshold plus the value of quantitative reasoning in the CSU's own courses. This is a new focus of responsibility and puts a range of concerns before us, detailed in the rationales and implementation notes for the recommendations below. We see this continuing to develop as the national discussion in quantitative reasoning progresses.

Quantitative Reasoning Task Force Recommendations

Recommendation I. Define Quantitative Reasoning.

The Task Force proposes this definition for quantitative reasoning in general:

> The ability to reason quantitatively is a stable combination of skills and practices involving: (i) the ability to read, comprehend, interpret, and communicate quantitative information in various contexts in a variety of formats, (ii) the ability to reason with and make inferences from quantitative information in order to solve problems arising in personal, civic, and professional contexts, (iii) the ability to use quantitative methods to assess the reasonableness of proposed solutions to quantitative problems, and (iv) the ability to recognize the limits of quantitative methods. Quantitative methods include the methods of computation, logic, mathematics, and statistics. Quantitative information is traditionally found in subject areas like mathematics, statistics, computer science, and logic.

Rationale for Recommendation I. The CSU does not have a definition of quantitative reasoning to guide planning and practice. This definition involves three important concepts: reasoning quantitatively, demonstrating general quantitative reasoning ability, and preparation for ongoing development of quantitative reasoning abilities.

It is based on, but differs from, those found in the following resources: What is Quantitative Reasoning: Defining the Construct for Assessment Purposes; Quantitative Reasoning for College Graduates: A Complement to the Standards of the Subcommittee on Quantitative Literacy Requirements of the MAA; AAC&U Quantitative Literacy Rubric; Assessing Quantitative Literacy in Higher Education: An Overview of Existing Research and Assessments with Recommendations for Next-Generation Assessment.[5]

The next section applies this definition to the different contexts in which students shall be required to demonstrate their ability to reason quantitatively.

[5] See Appendix D, Bibliography.

Recommendation II. Revise Quantitative Reasoning Requirements

Assessing the ability of students to reason quantitatively depends on their educational context. The quantitative reasoning definition proposed in Recommendation I is intended to inform revised policy (1) that evaluates the general quantitative reasoning ability of students entering and graduating from the CSU, (2) that articulates well with the CSU's sister segments (California public high schools, California Community Colleges, and the University of California), and (3) that specifies clear and implementable procedures for evaluating and improving general quantitative reasoning ability.

Such requirements must acknowledge that the world is changing and mathematics is changing along with it. The National Academies Report *Mathematical Sciences in 2025* made it clear that mathematics is broader than arithmetic, algebra, and calculus at the service of research mathematics, engineering and science.

> *The ongoing trend for the mathematical sciences to play an essential role in the physical and biological sciences, engineering, medicine, economics, finance, and social science has expanded dramatically. The mathematical sciences have become integral to many emerging industries, and the increasing technological sophistication of our armed forces has made the mathematical sciences central to national defense. A striking feature of this expansion in the uses of the mathematical sciences has been a parallel expansion in the kinds of mathematical science ideas that are being used.* (NAR - Math Sci 2025)

The current debate among mathematicians and the general public is whether a common quantitative reasoning set of skills and practices exists, and if so whether algebra has any part of it. Math requirements that prescribe intermediate algebra for everyone at the foundational level or college algebra for everyone at the college level have been described as "the single-file death march that leads towards calculus" (T. Holm, Boston Globe, February 2015). Nationally they are being replaced by pathways that are tailored to a student's major or career.

At the same time, algebra has also been called a "Civil Right" by Bob Moses. Similarly, Linda Rosen (CEO of Change the Equation) has stressed the importance of algebra in the workplace:

> *Corporate America understands that on-the-job-training will always be needed. Cutting-edge products and ideas inevitably require employees to learn new things. But, corporate America understandably balks at on-the-job-training that covers content that should have been learned — like algebra — before joining the workforce.*
>
> *Let's not throw the baby out with the bathwater. Instead, let's ensure that all students master algebraic thinking and problem-solving, the essence of algebra, regardless of their eventual career goals.* (Huffington Post, July 2012)

This speaks to a more practical view of the role of algebra in a student's development. It supports Nicholas Warner's (Physics, Math and Astronomy, USC) defense of algebra as part of a liberal arts education.

> *One of the less obvious goals in algebra is to get people to think more abstractly. Very elementary mathematics is all about "real things" and initially employs realia to help us add, subtract and multiply. From this experience we learn the language and some of the basic rules of mathematics. We abstract and generalize the experience and learn that, when we manipulate one side of an equals sign then the equality is only true if we do the same thing to the other side. Algebra makes a major intellectual leap: It names and labels things that we do not immediately know and that sometimes lie outside our direct experience. There are certainly other studies that involve abstractions like love, empathy and ethics, but in algebra we learn to handle abstractions that are not part of visceral human experience. We learn not only to be comfortable with such external unknowns but how to master them.* (Huffington Post, August 2012)

Such strong and seemingly divergent views of algebra's role in quantitative reasoning point to the urgency of the task to reconsider quantitative reasoning requirements and the role of algebra in them. Upon closer analysis, the views above need not be opposing sides of a controversy. Instead they call out for the need for a more subtle analysis of what quantitative skills and practices are truly necessary.

In making that evaluation, the Quantitative Reasoning Task Force referred back to its guiding principle: the need to balance access and opportunity to achieve equity. Each time a mandatory skill is added to the "baseline," we risk excluding students from the academy, and yet each time one is removed, we risk limiting the value of the degree pursued. The task is to define which quantitative skills practices give enough value that they are worth the risk of limiting access, and this must be done in a dynamic and changing world.

The Quantitative Reasoning Task Force sought a reasonable quantitative reasoning baseline on which additional specialized quantitative skills and practices could be built in the context of a student's interests, major, and intended career. The Task Force started with a logistical recommendation to separate the entry and exit level of quantitative reasoning.

Recommendation IIA: Separate foundational and baccalaureate quantitative reasoning requirements.

The Task Force recommends ending the use of prerequisite language to impose a de facto foundational quantitative reasoning requirement. Instead it recommends defining separate foundational and baccalaureate requirements that are reasonable and equitable.

Rationale for Recommendation IIA. The Quantitative Reasoning Task Force used the definition of quantitative reasoning in Recommendation I to guide its recommendations for quantitative reasoning policy. In doing so, the Task Force identified two perceived weaknesses of the current CSU quantitative reasoning policies.

- Current policy relies on “intermediate algebra as an explicit prerequisite” as the main identifier of a course that meets the B4 requirement. To move beyond this definition a well-articulated quantitative reasoning requirement is needed to provide a reasonable level of consistency between different CSU campuses, while maintaining principles of academic freedom.

- Serious inconsistencies exist between the Quantitative Reasoning requirements of native CSU freshmen and those of transfer students from community colleges. The inconsistencies may disproportionately and negatively impact historically underserved populations.

This rationale describes how the Task Force’s efforts to developed a well-articulated equitable quantitative reasoning requirement led to the proposed separation of the entry and exit quantitative reasoning requirements.

Articulating quantitative reasoning requirements from high school to community college to university requires balancing access and opportunity. As stated in the codified expectation section, current policy requires that any B4 (Quantitative Reasoning) course transferable to the CSU or UC “have intermediate algebra as a prerequisite.”

This statement is natural for a quantitative reasoning course taken by a student majoring in STEM (Science, Technology, Engineering and Mathematics) for whom the calculus pathway is mandatory. However, it does not make sense for the majority of students in the CSU who are taking statistics or quantitative reasoning courses to satisfy their general education requirement in quantitative reasoning. (See Table 1.) Such courses have greatly expanded in enrollment and content over the last 20 years, and the curriculum tends to be less algebraically intensive but in many respects significantly more conceptually challenging than intermediate or college algebra.

However, the Task Force members acknowledge that in the same 20 years the intermediate algebra threshold has served a secondary purpose as the de facto baseline of “foundational quantitative reasoning proficiency.” This baseline provided a common structure on which baccalaureate quantitative reasoning courses, as well as other general education courses, could be built. Removing that baseline or changing it may have serious consequences for students and programs. Many general education courses assume the content of intermediate algebra or the “mathematical maturity” that proficiency in intermediate algebra implies. Moreover, the growth in statistics and quantitative “life skills” general education courses was in part made possible by the foundational baseline because CSU faculty could be confident that students completing a General Education quantitative reasoning course would have demonstrated proficiency not only in the skills of that particular course but also in the more general skills of the foundational threshold.

It is interesting to note that in Assessing Quantitative Literacy in Higher Education: An Overview of Existing Research and Assessments with Recommendations for Next-Generation Assessment (ETS Research Report 2014) their proposed framework for assessing quantitative literacy in higher education is based on math content similar to the ELM. This suggests that deepening, extending, and contextualizing these skills is at the heart of college-level quantitative reasoning. This does not presuppose that students have

mastery of these skills prior to college or should be denied access to college based on this list of skills, but rather that these skills should grow and deepen over time.

The Quantitative Reasoning Task force researched national best practices, interviewed colleagues from STEM and non-STEM fields, and listened to presentations from policy makers and experts in the field:

- Ted Mitchell -- Under Secretary, US Department of Education
- Catherine Lhamon -- Assistant Secretary, Office for Civil Rights, US Department of Education
- Philip Daro - Mathematics educator and coauthor of the National Common Core Standards for Mathematics
- Bill McCallum -- University of Arizona Math professor and coauthor of the National Common Core Standards for Mathematics
- Robert Green -- UCLA Math professor and founding member of Transforming Post Secondary Education in Math
- Tristan Denley -- Vice Chancellor for Academic Affairs, Tennessee Board of Regents
- Estela Bensimon -- USC Higher Education Professor & Founder of The Center for Urban Education
- Christopher Edley -- Berkeley Law professor and President of The Opportunity Institute

The Task Force concluded that the current quantitative reasoning GE requirement, because it defines a quantitative reasoning course as one with "intermediate algebra as an explicit prerequisite," is a misrepresentation of current practice within the CSU, is a misuse of the word "prerequisite," and does not reflect current suggested best practices for undergraduate curriculum in mathematics and quantitative reasoning.[6]

The Task Force believes that separating foundational and baccalaureate quantitative reasoning benchmarks will create a more constructive environment within which requirements for both levels can be discussed. This separation allowed the Task Force to develop consensus definitions of quantitative reasoning requirements that balance access and opportunity.

The next recommendation, IIB, proposes a definition of quantitative reasoning at the baccalaureate level. The recommendation after that, IIC, proposes a definition of the foundational quantitative reasoning the CSU would expect of all students at entry.

Recommendation IIB: Define Baccalaureate Quantitative Reasoning

To earn a baccalaureate degree from the California State University, students shall (i) develop and demonstrate a proficient and fluent ability to reason quantitatively in a broad spectrum of the contexts defined by California Common Core State Standards for High School, (ii) develop and demonstrate a general understanding of how

[6] *De facto*, as reflected in the various GE curricula used across the CSU system, campus implementation of the current CSU quantitative reasoning requirement for graduation conforms to many of the suggested best practices for undergraduate students pursuing baccalaureate degrees in the US. As GE curricula vary across the 23 campuses within the CSU, the quantitative reasoning graduation requirements are implemented differently on different campuses.

practitioners and scholars solve problems quantitatively in a range of disciplines, (iii) develop and demonstrate an in-depth understanding of how practitioners and scholars solve problems quantitatively in a specialized area (e.g. the major), and (iv) be prepared to develop their ability to reason quantitatively after graduation in the various contexts defined by personal, civic, and professional responsibilities.

Rationale for Recommendation IIB. This definition reflects the existing good practice within the CSU in which students take quantitative reasoning B4 courses appropriate to their majors, general education interests, and careers. It also acknowledges that students develop quantitative reasoning outside of their B4 courses. Students have always reasoned quantitatively in general education classes in science, business, or technology, and are increasingly asked to do so as part of critical thinking on issues of equity, sustainability, and politics.

Recommendation IIB encourages systemwide conformity in the expected quantitative reasoning ability of students graduating from the CSU without infringing on academic freedom or being so prescriptive as to stifle the distinct campus cultures that thrive in the CSU. It is framed in the language of the Common Core State Standards and thus articulates well with our sister segments (California high schools, California Community Colleges, and the University of California). Finally, it specifies a framework within which clear and implementable procedures for evaluating and improving general quantitative reasoning can be assessed.

Notes on implementing Recommendation IIB. The view of the Quantitative Reasoning Task Force is that quantitative reasoning occurs throughout a student's academic program. In particular, it is not a proficiency achieved by taking one B4 course. The above requirement shall be managed through the existing processes that determine whether courses meet general education requirements. The B4 courses would provide the backbone of the quantitative reasoning skills while other general education classes that require quantitative reasoning (e.g. science) would deepen and broaden the student's practice. The Task Force noted that WASC has asked for upper division critical thinking or quantitative reasoning measures and Recommendation IIB lends itself to such development.

Of legitimate concern is how to identify college level quantitative reasoning courses that satisfy the Area B4 requirement for general education and graduation.

Within the CSU, courses that deepen or broaden students' quantitative reasoning significantly beyond that of the California Common Core State Standards for high school shall be deemed college level. For example, the typical course in Statistics would be college level whereas an Intermediate Algebra course would not be, since the content of Intermediate Algebra is completely contained within the Common Core State Standards. Moreover, Statistics would qualify not only as college-level, but also as a B4 course.

In contrast, a history class may use quantitative reasoning at the college level; however, it will be unlikely to develop student proficiency to the extent the course would meet the B4 criteria. The Task Force supports the development of a general rubric which can be adapted by CSU and community college campuses to evaluate courses against B4 criteria. The delicacy of these boundaries and the inevitable controversy they will cause emphasize the

need for continued dialogue and development, ideally to include faculty, evaluators, and articulation officers with guidance from a CSU Center. (See Recommendation IV.)

Recommendation IIC: Define Foundational Quantitative Reasoning

Upon entering the California State University in pursuit of a baccalaureate degree, students will be prepared to develop their ability to reason quantitatively in the broad spectrum of courses involving quantitative reasoning offered within the CSU (including, but not limited to, B4 courses). In particular, a student who has satisfied the Foundational Quantitative Reasoning requirement shall have:

- *Demonstrated proficiency and fluency* in the combined skills found in the Common Core State Standard curriculum for K-8, Algebra 1, and Integrated Math 1
- *Practiced* the skills in the high school curriculum in a variety of contexts that broaden, deepen or extend K-8, Algebra 1 and Integrated Math 1 skills.[7]
- Developed the eight common core mathematical practices as applied to the K-8, Algebra 1, and Integrated Math 1 skills, which are the abilities to:
    - Make sense of problems and persevere in solving them
    - Reason abstractly and quantitatively
    - Construct viable arguments and critique the reasoning of others
    - Model with mathematics
    - Use appropriate tools strategically
    - Attend to precision
    - Look for and make use of structure
    - Look for and express regularity in repeated reasoning.

Rationale for Recommendation IIC. While the Quantitative Reasoning Task Force found consensus fairly easily around the definition of the Baccalaureate Quantitative Reasoning requirement, the boundaries of the Foundational Quantitative Reasoning requirement (mastered and practiced) were more problematic, as their identification required looking at what quantitative reasoning preparation a student would need in a broad range of majors, general education interests, and careers as well as in civic life.

Moreover this definition relates the CSU to all segments of California's public education system, as illustrated in a number of possible scenarios:

- James is a high school junior whose test results indicate he is only "conditionally proficient" in Foundational Quantitative Reasoning. To satisfy the condition for full readiness, he would benefit from senior year course options to reach full proficiency for quantitative reasoning in the CSU.

- Samantha is a community college student hoping for an Associate Degree in Psychology. She did not graduate from high school. She needs a well-designed pathway or series of courses to achieve foundational and baccalaureate proficiency before transferring to the CSU. As much as possible this course work should relate to her major and interests.

[7] Including quantitative reasoning skills as practiced in high school curriculum outside of mathematics.

- Maura is a CSU entering biology major who is not proficient in Foundational Quantitative Reasoning. She needs some developmental math coursework to prepare her for pre-calculus.

- José is an entering sociology major who is not proficient in Foundational Quantitative Reasoning. He needs some developmental math coursework to prepare him for statistics.

The Foundational Quantitative Reasoning requirement needs to cope with this full spectrum of students and needs to support a broad range of non-algebra intensive majors, general education interests, and careers, while preparing students for civic life.

In trying to identify the correct threshold for the Foundational Quantitative Reasoning requirement, the Quantitative Reasoning Task Force members relied on multiple sources including the 2013 Report of California's Intersegmental Committee of Academic Senates ("ICAS"), the 2015 California State University Council of Math Chairs' Statement on Entry Level Mathematics and Statway, and evaluations of the California Common Core State Standards.

The Task Force found that the language of "mastered" and "practiced" (commonly used in secondary math standards) was very helpful in defining the Foundational Quantitative Reasoning threshold. It allowed the resulting requirement to raise the standard for proficiency on some basic topics while allowing coursework and other measures to stand in for measuring others. However, by calling for deeper learning for some essential concepts, the Task Force doesn't intend to recommend individual test instruments or any threshold scores (e.g. 80% or 90%) that may be implied by the word "mastery" in other sectors of education. Instead, the words "mastery" or "proficiency" are used colloquially, to describe internalized learning that students are prepared to apply confidently in a range of settings.

To get a broad and national view, Task Force members looked at reports from professional mathematics and statistics organizations, national studies, and leaders in STEM and non-STEM professions. (See Appendix D for a full bibliography.) The Quantitative Reasoning Task Force paid particular attention to majors that lead to careers in nursing, teaching, law enforcement, and business as these are typical non-STEM careers, which attract students who hope to move into the middle class. The mathematical skills and practices students would need for the quantitative reasoning required in those majors were expressed in terms of the California Common Core State Standards for mathematical skills and practice.

The Standards of Mathematical Practice, spelled out in the Common Core State Standards, provide a broad framework of habits of mind that when practiced in contexts requiring mathematical skills are *quantitative reasoning*. The mathematical skills of the CCCSS grow upon one another in the K-12 curriculum forming a tall, narrow tree of knowledge. In fact, this construct is central to the National Common Core Standards where skills are developed through just a few "progressions:" Number Systems, Expressions and Equations, Functions, Geometry, and Statistics and Probability.

> *In general, the Common Core's progressions resist the idea of mathematics as a list of topics because lists quickly become too long for students to keep in their active memories. Rather the progressions invite students to recognize underlying principles. This recognition "shrinks" the mental real estate required for memorization while deepening mathematical understanding.* (CIME, 2016)

Because the mathematical knowledge tree is narrow, defining Foundational Quantitative Reasoning means deciding which branches of the curriculum are fundamental to our purpose of student opportunity, while maintaining maximal access to higher education.

The Task Force looked for a Foundational Quantitative Reasoning threshold that would guarantee the mathematical skills necessary to student success in non-algebra intensive majors, quantitative reasoning skills for life typically taught in an "ideas in math" class, and a very narrow list of skills and knowledge that members considered necessary for a liberal arts education.

Statistics is a prototype non-algebra-intensive Baccalaureate Quantitative Reasoning course. Recent work suggests that in the context of the Common Core State Standards, to be successful in Statistics a student would need to be proficient in the most of the K-8 curriculum as well as several topics from the Algebra 1 or Integrated Math 1 curriculum. For example, a student needs to be able to evaluate algebraic expressions in order to calculate numerical summary statistics, test statistics, confidence intervals, z-scores and regression coefficients in Statistics. (Utts et. al., 2015)

Additionally, CSU graduates in any major will likely need to manage a business budget or choose among mortgage options. Thus, they should have the necessary skills to be ready to learn about personal and business financial models: simple and compound interest as well as the fundamentals of cost, revenue, and profit. This future learning might happen in a quantitative reasoning class, a GE elective on sustainability, or even on the student's own after graduation, but the foundations are necessary. Readiness to learn financial models requires further skills found in Algebra 1 or Integrated Math 1, For example, "Interpret functions that arise in applications in terms of the context" or "Construct and compare linear and exponential models and solve problems."

As a result of this analysis, the Quantitative Reasoning Task Force found that the correct Foundational Quantitative Reasoning requirement for mastered skills lies quite close to the combination of the K-8 plus the Algebra/Math 1 curriculum. This standard concurs with the conclusions of Georgia, Texas, Indiana, and Maryland and is close to the Entry-Level Mathematics threshold supported by the Council of CSU Math chairs. In particular, the ELM threshold does not require exponential models at all, but it does require students to manipulate expressions involving ratios. The Quantitative Reasoning Task Force feels that such distinctions can be readily reconciled via broad consultation over the 2016-2017 academic year. In many cases, it may be a matter of defining more specifically what level and depth is intended by the standards.

The Task Force strongly recommends that the CSU operationalize this definition of Foundational Quantitative Reasoning by drawing wherever possible from the California Common Core State Standards ("CCCSS").

However, the CSU is advised to monitor the impacts of this recommendation on student attainment and equity, and to continuously evaluate the connections drawn by CCCSS curriculum between skills and their justifications. For example, it is reasonable to say that students should be able to “evaluate algebraic expressions,” “compute compound interest,” or “be able to solve a linear equation in one variable” in a simple interest formula. However, it was the consensus of the Task Force that it would be unreasonable to require a student in a non-algebra-intensive field to solve for time in a compound interest formula, $A = P(1+r/m)^{mt}$, by using logarithms. The Task Force acknowledges that the proposed recommendation is just the one iteration in a series of refinements and alterations.

Implementation notes for Recommendation IIC. Just as with the current policies related to the ELM test, a standard for Foundational Quantitative Reasoning is not intended as a CSU admissions requirement for first time freshmen. Rather it is an expectation for entering students, which if not met at entry must be satisfied through developmental math course work under existing guidelines.

Any measure of demonstrating Foundational Quantitative Reasoning proficiency should include as a significant part of the measure a proctored assessment of the skills to be mastered for Foundational Quantitative Reasoning proficiency.

In the short term, the Foundational Quantitative Reasoning requirement could be implemented using the existing Smarter Balanced/SAT/ACT/ELM structure, although the thresholds of the SAT and ACT should be revised as they are based on the old Intermediate Algebra standards. The Quantitative Reasoning Task Force recommends that an implementation team review this Foundational Quantitative Reasoning recommendation in particular, with attention to feasibility, relevance, and equity in fall 2016. That team should recommend any necessary changes to the Smarter Balanced/SAT/ACT thresholds and possibly to the ELM content as determined by the CSU.

The Quantitative Reasoning Task Force recognizes that quantitative reasoning as employed in majors, careers, and civic life is an evolving construct. Thus, the Foundational and Baccalaureate Quantitative Reasoning requirements will need to be revisited regularly. The Quantitative Reasoning Task Force calls on the CSU to develop both a streamlined process and a schedule for revising the Foundational and Baccalaureate Quantitative Reasoning requirement, in a process that is evidence based and takes into account all segments of the California public education system, as well as national trends and consensus.

To that end, the Quantitative Reasoning Task Force calls upon the professional societies from both STEM and non-STEM fields to work with the Transforming Post Secondary Education in Mathematics organization (TPSE Math) to do an in depth study of the causal relationships between Common Core Math skills and practices and Baccalaureate quantitative reasoning skills and practices. This has already been done for Statistics (item 1 above) in collaboration with sociology and psychology, however, it should be done for "quantitative reasoning" classes and broadly for Meta-Majors (see Denley). Doing this work department by department, campus by campus, or system by system is neither desirable nor efficient. Such justification is a "big information" endeavor in which a broad range of experts and practitioners needs to be consulted. A piecemeal approach will render inconsistent

results and replicate work that should be shared. Once the work is done broadly, individual departments, campuses and systems can tailor the results to their own environments based on their students, resources, and academic goals. In particular, such work could be used at the time of the next review of the Foundational Quantitative Reasoning requirement.

Recommendation III. Adopt implementable policies that ensure equitable access and opportunity to all CSU students

Recommendation IIIA: Promote Equity, Access & Opportunity

Make policies equitable for transfer students and developmental math students. Provide reasonable access to quantitative reasoning courses that open up opportunities in students' majors, interests, careers, and civic lives.

Rationale for Recommendation IIIA. This recommendation speaks directly to the inequities described in the background section by calling on the CSU to change its policies so that transfer students and CSU first time freshmen requiring developmental math coursework are held to the same Foundational and Baccalaureate Quantitative Reasoning proficiency standards.

As a consequence of such policy changes, the Task Force encourages the CSU to ensure that:

- All CSU campuses shall provide students with at least one B4 course that has no prerequisites beyond the Foundational Quantitative Reasoning requirement. Such courses shall be relevant to students' majors and interests (e.g. statistics, ideas in quantitative reasoning, or mathematics for life).

- Students with algebra intensive majors, interests, and career goals may be required to take additional mathematics at either the baccalaureate or developmental level prior to taking the appropriate B4 course. (E.g. a student may need intermediate algebra or college algebra prior to taking pre-calculus or mathematical methods in business.)

Implementation notes for Recommendation IIIA. The CSU needs to develop rubrics or other means to determine whether successful completion of course, pathway, or sequence of courses should be sufficient to demonstrate Foundational Quantitative Reasoning proficiency.

The implementation of these two recommendations will require that they be discussed in the contexts within which students experience them. The next subsections discuss three common contexts and how all the recommendations so far should be implemented in each. The first context is high school, where one additional recommendation is made in support of a recent resolution of the Academic Senate CSU.

Recommendation IIIB: Require four years of high school quantitative reasoning.

The Quantitative Reasoning Task Force recommends that four years of high school quantitative reasoning coursework be required as part of the CSU admissions criteria (per ASCSU Resolution AS-3244-16/APEP).

Rationale for Recommendation IIIB. As the ASCSU noted in the rationale for AS-3244-16/APEP, the success of incoming students is maximized when students maintain their exposure to mathematics/quantitative reasoning. Similar to the ability to speak in a second language, mathematical skills decline with lack of use, and it is important that students continue practicing and developing quantitative abilities throughout their academic careers. In a number of settings, including the CSU Admission Handbook and through CSU Mentor, the CSU already recommends four years of mathematics,[8] even though only three such years are required. Additionally, the standing ICAS recommendation in the Statement on Competencies in Mathematics Expected of Entering College Students is:

> *For proper preparation for baccalaureate level course work, all students should be enrolled in a mathematics course in every semester of high school. It is particularly important that students take mathematics courses in their senior year of high school, even if they have completed three years of college preparatory mathematics by the end of their junior year. Experience has shown that students who take a hiatus from the study of mathematics in high school are very often unprepared for courses of a quantitative nature in college and are unable to continue in these courses without remediation in mathematics.*[9]

It is important to note that the fourth-year mathematics course called for by the CSU resolution would not necessarily be a fourth course in Area c; it must be a-g compliant, but it could be a course approved in Area g.

Other states in the US already require a fourth year of mathematics for admission to their state university systems. For example, effective with the class that entered this year (Fall 2015), students in Maryland will be required not only to complete four years of mathematics for entry to any of the Maryland public universities, but students who complete Algebra II prior to their final year must complete the four-year mathematics requirement by taking a course or courses that utilize non-trivial algebra.[10] The Maryland policy was based in part on the report, Coming to Our Senses: Education and the American Future,[11] which found that the academic intensity of high school curriculum was the most important predictor of college success and which recommended four years of college preparatory mathematics.

These findings and prescriptions are not new. In "Overcoming the High School Senior Slump: New Education Policies," Kirst argued in 2001[12] that to reclaim the senior year, high schools should redesign their senior year courses to serve as gateways to general education requirements that students would encounter in their first year of college and emphasize the

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[8] http://www.csumentor.edu/planning/high_school/subjects.asp
[9] http://icas-ca.org/Websites/icasca/images/ICAS-Statement-Math-Competencies-2013.pdf)
[10] https://www.washingtonpost.com/local/education/heading-off-the-senior-slump-maryland-schools-officials-to-require-fourth-year-of-math/2014/05/02/7458986e-c9b4-11e3-a75e-463587891b57_story.html; http://www.usmd.edu/newsroom/news/1021; https://www.admissions.umd.edu/requirements/Freshmen.php; http://undergraduate.umbc.edu/apply/freshmen.php
[11] http://media.collegeboard.com/digitalServices/pdf/advocacy/admissions21century/coming-to-our-senses-college-board-2008.pdf
[12] http://files.eric.ed.gov/fulltext/ED455720.pdf

importance of taking senior-year math courses, and that colleges should include a senior-year math course in their admissions requirements.

There is a strong correlation between taking more mathematics in high school and being college-ready upon arrival at the university. This can be seen both in terms of

1. Studies of how SAT-Math and ACT-Math scores improve as the number of years of high school mathematics increases[13] and
2. Studies of how the likelihood of needing remediation decreases and the likelihood of completing general education Quantitative Reasoning requirements increases as students take more high school mathematics.[14]

Finally, many former high school students, with the clarity of 20-20 hindsight, recognize that they should have taken more (or more difficult) mathematics courses in high school. A one-year later survey of 1507 high school graduates found that 44% wish they had taken different courses in high school. The most frequently expressed regret (40% of this group, or more than one in every six students surveyed) was that students hadn't taken more or higher level mathematics courses.[15]

Additional background is provided in Appendix E, Additional Rationale for 12th Grade Quantitative Reasoning.

Implementation notes for Recommendation IIIB. If the CSU adopts this admission requirement, there will be a natural implementation phase of at least three to four years. The CSU cannot impose this requirement on students already enrolled in high school; it will be operational only as the next 8th grade class enters the 9th grade. With this in mind, the CSU needs to move forward by communicating its intention to all stakeholders and interested parties as soon as possible.

The CSU will be in a better position to assist high schools in meeting the new requirement with existing Area C and appropriate Area G courses as well opportunities for professional development if the system supports creation of a Center for the Advancement of Instruction in Quantitative Reasoning. The Center would be charged with developing a modular course patterned after the Expository Reading and Writing Course, which was designed to reduce remediation needs in English.

Many California high schools already offer such a 12th grade course in quantitative reasoning. More than 60 percent of students advancing to the CSU from high school already are completing four years of math. The goal is to fill in the gap and overcome what may be a two-year absence in the use of acquired quantitative skills for students.

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[13] https://secure-media.collegeboard.org/digitalServices/pdf/sat/total-group-2015.pdf; https://secure-media.collegeboard.org/digitalServices/pdf/sat/TotalGroup-2014.pdf; http://media.collegeboard.com/digitalServices/pdf/research/2013/TotalGroup-2013.pdf)

[14] http://higheredutah.org/high-school-math-critical-to-completion/

[15] http://media.collegeboard.com/homeOrg/content/pdf/One_Year_Out_key_findings%20report_final.pdf

How students satisfy the requirement for 12th grade quantitative reasoning would depend on the student's proficiency upon entering the senior year. It could be an a-g course that introduces new material, or the course could reinforce learning from earlier years.

High School Quantitative Reasoning Course Definition: If the a-g required course work in math is being completed in the senior year, such as Algebra II or Integrated Math III, then this course will count as the student's fourth year of quantitative reasoning. If the a-g required course work in math is being completed in the junior year, then the student must take math-based quantitative course work in the senior year. This requirement may be met in many ways:

- By completing an advanced level math course (pre-calculus, math analysis, calculus)
- By completing an Area c or g course in statistics, quantitative reasoning, mathematics or computer science or any other approved math-based quantitative Area c or g course.
- By completing an algebra-based Area d science course (chemistry, physics).

In California, the Common Core State Standards determine what students in grades K-12 should know and be able to do in mathematics, and the Smarter Balanced Assessment is used to assess attainment of the standards. Any CSU-admissible student is required to complete the full CCCSS K-12 curriculum and as a result will be deemed to have completed the parts of the Foundational Quantitative Reasoning requirement that state that a student shall have "practiced the skills in the high school curriculum" and "have developed the eight common core mathematical practices as applied to the K-8, Algebra 1, and Integrated Math 1 skills."

What remains is to determine whether a student has "Demonstrated proficiency and fluency in the combined skills found in the Common Core State Standard curriculum for K-8, Algebra 1, and Integrated Math 1." As stated earlier, Title 5 requires that the CSU identify "as quickly as possible" those admitted students "who cannot demonstrate… such basic competence" and require them to engage in what is commonly called remediation.

The junior year Early Assessment Program and Smarter Balanced Assessment results are the means for informing CSU-bound students of their quantitative reasoning status "as quickly as possible" (Title 5). The CSU designates entering students as proficient, conditionally proficient, or not proficient in quantitative reasoning for purposes of preparation for CSU baccalaureate. By learning their proficiency status a year before they graduate from high school, CSU-bound students can proactively use their senior year to engage in quantitative reasoning coursework to help them attain proficiency prior to admission.

Below are three statements of proficiency designations and recommendations. (Note that we use the term "CSU math-eligible" to mean that a student has not only met the mathematics admission requirements to the CSU but is also ready for college-level work.)

For purposes of the recommendations below, the assumption is that Recommendation IIIA will be implemented. That is, in their senior year, students should enroll in a quantitative reasoning course as determined by their junior year Smarter Balanced Assessment

proficiency status in order to reduce or eliminate the need for developmental math course work in the CSU and at participating California Community Colleges.

- **Foundational Quantitative Reasoning *Proficient* Students.**
  - These students shall take any high school quantitative reasoning class as a senior.
  - They will be CSU math-eligible and will not require developmental math at the CSU or at any participating California Community Colleges.

- **Foundational Quantitative Reasoning *Conditionally Proficient* Students.**
  - These students shall take an Area c or appropriate Area g high school quantitative reasoning course.[16] Alternatively, such students may take any quantitative reasoning high school course in conjunction with a CSU-approved method for determining Foundational Quantitative Reasoning proficiency.
  - Students who pass the Area c high school quantitative reasoning course or are deemed Foundational Quantitative Reasoning proficient via another CSU-approved method shall not be required to enroll in developmental math at the CSU or at any participating California Community Colleges.

- **Foundational Quantitative Reasoning *Not Proficient* Students.**
  - These students shall take any high school quantitative reasoning course (however, Area c or g is recommended) in conjunction with a CSU-approved method for determining Foundational Quantitative Reasoning proficiency.
  - Students deemed Foundational Quantitative Reasoning proficient via any CSU-approved method shall not be required to enroll in developmental math at the CSU or at any participating California Community Colleges.

The definitions of Proficient, Conditionally Proficient and Not Proficient are based on the current practice of designating students as such, and date back to the inception of the Early Assessment Program (EAP) in the CSU. The change proposed in this recommendation is the additional requirement of a senior year math class.

As discussed above, the implementation of fourth-year math classes and the attendant proficiency protocol is an ambitious endeavor—one that will take time, collaboration, resources, and most importantly an attention to equity. The Task Force recommends that the timeframe to implement this requirement be extended far enough to allow high schools the needed time to develop capacity. It further recommends that the CSU and CCC partner with high schools and create a Center charged with developing appropriate curricula, assessing the outcomes of that curricula, and using the evidence to inform revisions of the curricula.

[16] This represents an expansion of the options for students to fulfill the conditional exemption with appropriate area G courses instead of only Area C courses. An AP computer science course could qualify in this category.

Recommendation IIIC: Ensure Early and Appropriate Quantitative Reasoning Courses for CSU First-Time Freshmen

The Task Force recommends reevaluating Quantitative Reasoning requirements in the context of the student's educational goals and proficiency at entry.

First time freshmen who enter the CSU shall be designated as proficient, or not proficient in Foundational Quantitative Reasoning based on academic record. First, as cohorts of entering students include more whose whole primary and secondary educations were under the California Common Core State Standards, any CSU-admissible student shall have completed the full CCCSS K-12 curriculum and as a result will be deemed to have completed the parts of the Foundational Quantitative Reasoning requirement that state that a student shall have "practiced the skills in the high school curriculum" and "have developed the eight common core mathematical practices as applied to the K-8, Algebra 1, and Integrated Math 1 skills."

What remains is to determine whether a student has "Demonstrated proficiency and fluency in the combined skills found in the Common Core State Standard curriculum for K-8, Algebra 1, and Integrated Math 1" As mentioned above, this designation currently is done in a series of steps starting with the Smarter Balanced Assessment in math in the junior year. In the senior year, students take the SAT and/or the ACT and either test can qualify a student as proficient. Finally, if at the end of his or her senior year a student is still not proficient then the student takes the ELM exam in spring after being admitted to the CSU. The ELM exam results indicate whether the student is proficient or not.

Under current rules not proficient students must begin their developmental math preparation the summer before enrolling for the fall semester of their first year at the CSU. These students must complete all developmental math work by the end of their first year in the CSU. Failure to do so results in the student being "stopped out" of the CSU until he or she completes an intermediate algebra course at a community college or via testing. In practice, the stop-out policy is not uniformly enforced across the CSU and should be reviewed.

The Quantitative Reasoning Task Force recommends the following for incoming first time freshmen in the CSU:

- **Foundational Quantitative Reasoning Proficient** students shall take a baccalaureate quantitative reasoning class in the first two terms at the CSU. Options shall exist in the context of the student's major and interests.

- **Foundational Quantitative Reasoning Not Proficient** students shall demonstrate Foundational Quantitative Reasoning proficiency within two terms of enrollment via a CSU-approved method for determining Foundational Quantitative Reasoning proficiency. They shall take a baccalaureate quantitative reasoning class within two semesters of demonstrating Foundational Quantitative Reasoning proficiency. Options shall exist in the context of the student's major and interests. This recommendation is intended to accommodate co-requisite remediation, at the option of the institution providing the instruction.

Rationale for Recommendation IIIC. As pointed out in Recommendation IIIB, students in algebra intensive fields like STEM or business may be required to take additional mathematics at either the college or developmental math level. This presents an interesting challenge for developmental math grades, as illustrated in the following scenarios:

- Maura is a CSU entering biology major who is not proficient in Foundational Quantitative Reasoning. In fact she requires two semesters of developmental math work.
  - In her Summer Early Start math class she is not able to apply herself fully because she is working 40 hours per week as a pharmacy checkout clerk. She makes sufficient progress to fulfill the Early Start requirement but does not improve her fall math placement.
  - In fall, she receives credit in Developmental Math 1 For Algebra-Intensive Majors. (This is a new category of developmental math course, proposed as part of this Recommendation IIIC. Maura would be enrolled in it because biology is considered an algebra-intensive major.)
  - In Spring, she makes progress but not enough to earn credit in intermediate algebra. However, her average over the course of the semester does indicate that she is proficient in Foundational Quantitative Reasoning.

- James is a CSU entering sociology major who is not proficient in Foundational Quantitative Reasoning. In fact, he too requires two semesters of developmental math work.
  - In his Summer Early Start math class, he is not able to apply himself fully because he working 40 hours per week as a receptionist in a health clinic. He makes sufficient progress to fulfill the Early Start requirement but does not improve his fall math placement.
  - In fall, he receives credit in Developmental Math 1 For Non-Algebra-Intensive Majors.
  - In Spring, he earns credit in Developmental Math 2 For Non-Algebra-Intensive Majors, a class that teaches no more content that is necessary for proficiency in Foundational Quantitative Reasoning.

In terms of content James and Maura may be comparable from the perspective of Foundational Quantitative Reasoning. Neither one should be stopped out. However, a grade of “credit” in Maura’s class would falsely depict her as ready for pre-calculus or college algebra. For such a student an alternate grade to the traditional credit and no credit is required. One model could be a grade of “P” could be used to denote that the student has demonstrated proficiency in Foundational Quantitative Reasoning. Such a grade leaves Maura, the biology major, with a choice: either switch to a major requiring a non-algebra intensive coursework, or remain a biology major and repeat intermediate algebra.

Recommendation IIID: Establish equitable articulation of quantitative reasoning credit for transfer students.

Community College students should be assessed by the community colleges as proficient or not proficient in Foundational Quantitative Reasoning in alignment with the standards above.

Prior to transfer, they should demonstrate Foundational Quantitative Reasoning proficiency and earn the appropriate minimum grade in a course that transfers for B4 credit.

Such a student will not necessarily be considered proficient in Baccalaureate Quantitative Reasoning as certain campuses may require upper division work for this designation. Articulation for Foundational Quantitative Reasoning proficiency will follow the existing approval process for B4 transfer approval. The Task Force supports the creation of options for both Foundational and Baccalaureate Quantitative Reasoning that teach skills and practices in the context of the student's major and interests.

Implementation notes for Recommendation IIID. To provide more equitable access to the CSU and to ensure that students are ready for the rigors of baccalaureate work, the Task Force has replaced intermediate algebra requirements with a Foundational Quantitative Reasoning requirement. To meet the needs of all community college students who plan to transfer to the CSU, these new standards may require new approaches.

Students who are not deemed proficient in Foundational Quantitative Reasoning by the community college assessment process will need opportunities to obtain these skills prior to transferring to the CSU. These opportunities may be embedded in, or taught as a co-requisite for, a B4 transfer level quantitative reasoning course or achieved in separate course work. Coursework designed to address the foundational quantitative reasoning requirement should provide opportunities for students to deepen and broaden quantitative reasoning skills in a wide variety of contexts from the K-12 curriculum, as well as frequent opportunities to engage in learning experiences that promote the CCCSS mathematical practices.

The Task Force supports initiatives to ensure more equitable ways to bring post-secondary education to California's students by creating new quantitative reasoning pathways (such as efforts of the Carnegie Foundation for the Advancement of Teaching and the California Acceleration Project). The revised quantitative reasoning requirements, which bring the official position of the CSU much closer to the curricula developed in these pathways, are intended to give guidance for developing such innovations and therefore eliminate the need for further exceptions and waivers.

Recommendation IV. Create a CSU Center for the Advancement of Instruction in Quantitative Reasoning

As soon as possible, the CSU should create a Center for Advancement of Instruction in Quantitative Reasoning to act on the Task Force's and subsequent findings, and to continuously support the high quality instruction and curriculum in the high schools, community colleges, and public universities that will better serve the state.

Rationale for Recommendation IV. The Task Force appreciates the rapidly changing contexts of high school instruction, best practices in postsecondary education, and the skills in quantitative reasoning that CSU students will rely on after graduation. There is a need for ongoing, consistent and coherent oversight and coordination of statewide efforts to make progress in mathematics education.

Recommendations IIIA through IIID propose profound changes to policy, that would rely for their implementation on deeper and more sustained partnerships with CSU colleagues in California's public schools, community colleges, and the University of California. The all-purpose label "intermediate algebra" almost certainly conveyed a false sense of sequential learning in quantitative reasoning, while exacerbating disturbing inequities across the state. But it had the virtue of being unambiguous: once faculty had set the ground rules, day-to-day operation could be delegated to others.

By contrast, a more equitable, sophisticated and responsive expectation for quantitative reasoning at entry and graduation will be harder to outsource. Instead, the CSU will need to take meaningful, responsible action to replace the label "intermediate algebra" with ongoing involvement in determinations of readiness at entry and transfer.

The Task Force believes that its recommendations are an important step toward such committed participation. The CSU has the capacity to bring to scale a more defensible set of benchmarks for student attainment, informed by the Common Core State Standards, bolstered by a universal expectation for quantitative reasoning in the 12th grade, and then developed at the baccalaureate level in ways that are fair for students of all backgrounds in both the CSU and community colleges.

The Center could also be an important source of intersegmental professional development, and research into student flow across California's educational sectors, giving faculty the means to monitor and adjust the definitions of foundational and baccalaureate quantitative reasoning proposed here.

Implementation notes for Recommendation IV. The model for this Center is the CSU Center for the Advancement of Reading, which for ten years has led development and deployment of a 12th grade Expository Reading and Writing and Course ("ERWC") across the state. The ERWC has been nationally recognized for its success improving college readiness in English, a track record that most observers ascribe to three factors in particular:

1. Stable, central administration of courses that nonetheless benefit from local innovation and customization.

2. Continuous development and refinement of curriculum, not just at the 12th grade level but also leading up to it, with scaffolded modules that begin as early as middle school.

3. Built-in professional development for high school teachers.

The CSU Center for Advancement of Instruction in Quantitative Reasoning would be built on the same principles, in the belief that student proficiency will be improved not by more exposure to advanced or esoteric topics in math, but by deeper and more varied practice in the concepts already learned.

However, to the core mission of the Center for the Advancement of Reading, the Center for the Advancement of Instruction in Quantitative Reasoning would add critical oversight and guidance for CSU and community college educators seeking to teach quantitative reasoning at the baccalaureate level. The Task Force believes the CSU's own Colleges of Education and Math Council will provide critical guidance for this work, and important capacity to follow through as they educate the next generation of math teachers.

Across the course of its research, the Task Force heard this emphasized by nearly every one of its advisors and in the literature: CSU students don't need more math at entry, and for many majors don't need more math later, either. Instead, they need more proficiency in the math they already have. Requiring a fourth year of quantitative reasoning in high school, and calling on the colleges and universities to broaden their conception of quantitative reasoning, will succeed only if such a Center can also be created, to ensure that the additional learning is in fact deeper instead of broader.

Topics for Further Study

A. The Task Force urges the CSU to conduct further studies on the use of "multiple measures" of college readiness in quantitative reasoning (for example, using proficiency as measured by high school grades in addition to single-administration test measures such as the SAT or ACT). A significant finding of the Task Force was that current policy, by treating all quantitative reasoning as sequential and relying on standardized testing as the main measure of readiness, may have disparate impacts on students from diverse backgrounds or who begin at community colleges. In particular, an updated reliability and efficacy study should be done on the ELM test. Also, data should be analyzed to determine correct SAT and ACT threshold scores for Foundational Quantitative Reasoning Proficiency.

B. Soon after its formation the Center should bring together (1) faculty in math and other quantitative disciplines and (2) representative staff in admissions, testing, evaluation, and articulation, and (3) educators at the high school level, who can develop rubrics for the determination of proficiency at entry and transfer.

C. The Center should lead development of a quantitative reasoning course in the 12th grade analogous to the Expository Reading and Writing Course (ERWC) for high school seniors (Quantitative Reasoning-HS) in area c or g. Such development should be informed by the numerous very encouraging examples of such courses locally around the state in high school and postsecondary partnerships.

The new, state-level course should be made available to high school teachers in modules that apply the skills and practices to be mastered in Algebra/Math I and many that are introduced and practiced in the full Common Core State Standards K-12 curriculum. Importantly, the course should have a strong focus on preparing students to engage in quantitative reasoning across a wide range of majors, interests, and careers, including, but not limited to teaching, nursing, law enforcement, information technology, sustainability, liberal studies, and social sciences.

Robust CSU faculty involvement in course development and high value professional development for faculty and high school teachers involved in implementation were two prominent features of the ERWC project. We call for the same in any forthcoming Quantitative Reasoning-HS model and rollout. We also recommend that the CSU establish a permanent position and Quantitative Reasoning Board to oversee Quantitative Reasoning across the CSU as well as issues of articulation and professional development.

Given the recent (May 2016) ASCSU resolution calling for the establishment of a center for mathematics instruction, such a center may be the appropriate home for development and oversight of the project. (See Appendix F, Academic Senate CSU Resolution 3253-16.)

D. Development and implementation of an upper division critical thinking assessment that combines quantitative and expositional reasoning.

Appendices

A. Academic Senate CSU Resolution 3230-15

ESTABLISHING A TASK FORCE ON THE REQUIREMENTS OF CSU GENERAL EDUCATION (GE) MATHEMATICS / QUANTITATIVE REASONING (B4) CREDIT

RESOLVED: That the Academic Senate of the California State University (ASCSU) appoint a task force to address two fundamental questions.

a. Can the pre-requisite content for the CSU GE B4 course be met concurrently with achieving the CSU GE B4 standards?
b. What should be the pre- (potentially co-) requisite content for quantitative reasoning and mathematical competency (CSU GE B4)?[17]

; and be it further,

RESOLVED: That the ASCSU define the membership of this task force to potentially include:

a. A member of the General Education Advisory Committee (GEAC) Statway advisory group
b. Another member of GEAC
c. A member of Academic Affairs (AA) Committee
d. A member of Academic Preparation & Education Programs (APEP) Committee
e. A representative of the Math Council
f. A faculty member who teaches B4 outside of mathematics
g. A California Acceleration Project (CAP) or Statway instructor
h. A member of the Entry Level Mathematics (ELM) test development committee
i. A representative of the CSU Office of the Chancellor
j. A representative of the Academic Senate of the California Community Colleges (ASCCC)
k. Any other interested ASCSU faculty member

RESOLVED: That the ASCSU distribute this resolution to University of California (UC) Board of Admissions and Relations with Schools (BOARS) leadership, General Education Advisory Committee (GEAC), CSU Math Council, Academic Senate of the California Community Colleges (ASCCC) Leadership, Executive Vice Chancellor Loren Blanchard.

***RATIONALE**: Five years ago the Chancellor's Office General Education Advisory Committee (GEAC) approved a limited pilot program within the California Community Colleges (CCC) in order to assess the viability of meeting CSU GE B4 quantitative reasoning requirements with a two-course integrated statistics sequence. This sequence bypasses the existing intermediate algebra proficiency in quantitative reasoning required by Executive Order (EO) 1100 as a prerequisite to CSU GE B4 courses. At its*

[17] Executive Order 1100 specifies Intermediate Algebra; the math council statement advocates for ELM content; Statway includes a lesser amount of algebra.

September 2015 meeting GEAC agreed to extend the pilot (at seven CCC districts) for an additional three years and invited other CCC districts to submit proposals utilizing curricular innovations in statistical pathways. In addition, GEAC called for the establishment of a task force to include disciplinary experts to review existing B4 standards in light of the fact that some of these statistics based pathways did not include a requirement to demonstrate proficiency in intermediate algebra prior to the award of B4 GE credit.

General Education Curricular standards are the province of the faculty and an expansion of the pilot has implications for CSU admissions and graduation standards and thus will rely on ASCSU action. The potential expansion of the GEAC pilot project on integrated statistical pathways for underprepared students generates a need to view the potential consequences of systemic changes to admissions standards and to EO 1100. Any potential changes will influence the minimum requirements for granting of a degree from the CSU.

Reducing achievement gaps and improving student success in meeting pre-baccalaureate and CSU GE mathematics/quantitative reasoning (B4) requirements are currently problematic. The traditional developmental pathway often constitutes a "leaky pipeline" in terms of success. As a result many students will never qualify for transfer because they cannot complete the prerequisites to CSU GE B4 requirements. Integrated statistical pathway programs such as the Statway pilot and the California Acceleration Project, were established to increase the number of community college students who would satisfy the CSU GE B4 requirement. There exists early work that illustrates the effectiveness of integrated statistical pathways (e.g., Carnegie Statway, California Acceleration Project, etc.) in reducing achievement gaps and improving student success as measured by pass rates. These efforts, however, do not achieve the levels of proficiency in intermediate algebra that are currently required for CSU freshman admission and thus introduce the specter of a "lesser degree" via lowering of academic standards.

The CSU Math Council, in their statement of April 2015, advocates that all students, at a minimum, attain knowledge of content as defined by the ELM requirements prior to the award of CSU GE mathematics/quantitative reasoning (B4) requirements. The statement reads in part:

> *We oppose the replacement of elementary or introductory statistics courses at CSU campuses by any program or pathway course lacking an explicit prerequisite or co-requisite* that subsumes the content of ELM. Such pathway courses include Statway. While the statistics content of Statway is totally aligned with the standard curriculum in elementary statistics, the pre-college mathematical content of Statway by itself does not meet the ELM standards and does not prepare students for college level courses. Hence Statway in its present form does not satisfactorily accomplish remediation and GE QR [quantitative reasoning/B4] in a single track, thereby pointing to the need of having all ELM content in a prerequisite or co-requisite*.*

There are unresolved discrepancies among the prerequisite B4 requirement (currently "Intermediate Algebra," per EO 1100); the potential use of ELM content (per the Math Council Statement); and the absence of any such pre/co-requisites for the CSU-approved Statway pilot project (and potentially other CSU-approved projects). This resolution attempts to address these concerns.

On the question of whether or not the pre-requisite knowledge could be achieved concurrently with the other B4 requirements, the answer is likely "yes" given the existence of "stretch" courses in which the content of a single course is stretched over multiple terms to allow inclusion of pre-baccalaureate material. It remains an open question whether or not the current pre-requisite (possible co-requisite) content should be Intermediate Algebra (per EO 1100), the material covered by the ELM exam (per the Math Council statement), or another standard (per "just in time" delivery of algebra via Statway).

A related issue of whether CSU GE B4 standards themselves could be satisfied by meeting one of two pathways (possibly Science, Technology, Engineering & Mathematics – STEM - vs. non-STEM; quantitative-based vs. statistically-based; etc.) should also be addressed once the issues touched on by this task force have been resolved.

Useful Definitions and Contextualization:

Title 5 requires "inquiry into mathematical concepts and quantitative reasoning and their applications." (CCR § 40405.1).

EO 1100 further explicates "Courses in subarea B4 shall have an explicit intermediate algebra prerequisite, and students shall develop skills and understanding beyond the level of intermediate algebra. Students will not just practice computational skills, but will be able to explain and apply basic mathematical concepts and will be able to solve problems through quantitative reasoning."

> ***§ 40402.1. Entry-Level Learning Skills.***
>
> *Each student admitted to The California State University is expected to possess basic competence in the English language and mathematical computation to a degree reasonably expected of entering college students. Students admitted who cannot demonstrate such basic competence should be identified as quickly as possible and be required to take steps to overcome the deficiencies. Any coursework completed primarily for this purpose shall not be applicable to the baccalaureate degree.*

Note: Authority cited: Section 89030, Education Code. Reference: Section 89030, Education Code.

***Attachments**: Math Council Statement; GE Guiding Notes excerpts on B4*

Approved Unanimously – September 4, 2015

B. Task Force Membership

Co-Chairs

Steven Filling
Professor of Accounting
CSU Stanislaus

Katherine Stevenson
Professor of Mathematics
CSU Northridge

Drafting Members

David Barsky
Associate Professor of Mathematics
CSU San Marcos

Bill Eadie
Professor of Journalism & Media Studies
San Diego State University

Denise Fleming
Professor of Education
CSU East Bay

Catherine Nelson
Professor of Political Science
Sonoma State University

Mark Van Selst
Professor of Psychology
San José State University

Mark Wheeler
Professor of Philosophy
San Diego State University

Non-Drafting Members

Keric Ashley
Deputy Superintendent
California Department of Education

Carolina Cardenas
Director, Academic Outreach Program
CSU Office of the Chancellor

Joey Freeman
Chief Policy Consultant
Lt. Governor's Office

Robyn Hines
Senior Director, State Governmental Affairs
Microsoft

Monica Lin
Associate Director of Undergraduate Admissions
UC Office of the President

Gavin Newsom
Lt. Governor of California
State of California

Ken O'Donnell
Senior Director, Student Engagement
CSU Office of the Chancellor

Ali Partovi
Co-Founder
Code.org

Henry Sanchez
Professor of Pathology and Medical Education
UC San Francisco

Myra Snell
Professor of Mathematics
Los Medanos College

John Stanskas
Professor of Chemistry
San Bernardino Valley College

Pam Walker
Vice Chancellor of Educational Services
CSU Office of the Chancellor

Laura Wallace
Professor of Mathematics
San Bernardino State University

Advisors

Stephanie Biagetti
Professor of Education
Sacramento State University

Pamela Burdman
Consultant
Independent Policy Analyst

Zee Cline
Executive Director
California Academic Partnership Program

Phil Daro
Consultant

Mark Ellis
Professor of Secondary Education
CSU Fullerton

Eric Forbes
Assistant Vice Chancellor, Student Academic Support
CSU Office of the Chancellor

Grant Fraser
Professor of Mathematics
CSU Los Angeles

Steven Graves
Professor of Geography
CSU Northridge

Mary Legner
Professor of Mathematics
Riverside City College

Virginia May
Professor of Math
Sacramento City College

Liliane Metlitzky
Professor Emerita, Mathematics
California State Polytechnic University, Pomona

Erik Shearer
Professor of Design
Napa Valley College

Julie Spykerman
Math Curriculum Specialist
Anaheim Union High School District

Ed Sullivan
Assistant Vice Chancellor, Academic Research & Resources
CSU Office of the Chancellor

Steven Wood
Professor of Criminal Justice
CSU Stanislaus

C. Course and Enrollment Data

The course and enrollment data cited in this report comes from these sources:

California High School Courses in Area C: Advanced Mathematics
Source: University of California Office of the President
Data Current as of April 15, 2016

California High School Courses in Area G: Electives with Quantitative Reasoning
Source: University of California Office of the President
Data Current as of June 14, 2016

California Community College Courses Approved for Transfer Credit in B4
Source: ASSIST Coordination Site, with invited corrections from colleges
Data Current as of June 17, 2016

California State University Courses in Area B4 of the GE Breadth Curriculum
Source: CSU Office of the Chancellor, with invited corrections from universities
Data Current as of June 17, 2016

The original records as provided to the Task Force are available for download in an Excel workbook, posted with this report under “Student Preparedness/Success” at http://www.calstate.edu/AcadSen/Records/Reports/index.shtml.

D. Bibliography

American Association of Community Colleges 2016. “Seizing the Moment: Community College Collaborating with K-12 to Improve Student Success.” Accessed May 11, 2016. http://www.aacc.nche.edu/newsevents/pressreleases/Documents/HEfHS-CommunityCollege-Paper-Final-web.pdf

Academic Senate of the California State University. 2015. “Establishing a Task Force on the Requirement of CSU General of CSU General Education (GE) Mathematics/Quantitative Reasoning (B4) Credit.” Accessed May 4, 2016. https://www.calstate.edu/acadsen/Records/Resolutions/2015-2016/documents/3230.shtml

Academic Senate of the California State University. 2016. “Support for Requirement a Fourth year of Mathematics/Quantitative Reasoning for Admission to California State University.” Accessed May 3, 2016. https://www.calstate.edu/acadsen/Records/Resolutions/2015-2016/documents/3244.shtml

Aldredge, R. 2015. “Statement on Approval of Statway Board of Admissions and Relations with School.” *University of California.* Accessed May 4, 2016. http://senate.universityofcalifornia.edu/committees/boars/documents/BOARSStatementonStatway.pdf

Aldredge, R. 2016. “Response Letter to Lieutenant Governor Newsom Computer Science Proposal. Board of Admissions and Relations with Schools.” *University of California.* Accessed May 4, 2016. http://senate.universityofcalifornia.edu/committees/boars/documents/BOARSResponsetoHon.Lt.GovernorGavinNewsomregardingcomputer-sciencecourses.pdf

AP Comparative Government and Politics 2015 Free-Response Questions. 2015. “The College Board.” Accessed May 12, 2016. https://secure-media.collegeboard.org/digitalServices/pdf/ap/ap15_frq_comp_gopo.pdf\

AP Environmental Science 2015 Free-Response Questions. 2015. “The College Board.” Accessed May 12, 2016. https://secure-media.collegeboard.org/digitalServices/pdf/ap/ap15_frq_environmental_science.pdf

Association of American College and University. 2016 “Quantitative Literacy Value Rubric.” Accessed May 12, 2016. https://www.aacu.org/value/rubrics/quantitative-literacy

Blake, M. 2016. “Gov. Brown proposes competition to create new high school math course.” *EdSource*. Accessed May 3, 2016. http://edsource.org/2016/gov-brown-proposes-competition-to-create-new-high-school-math-course/94051

California State University Council of Math Chairs’ Statement on Entry Level Mathematics and Statway. 2015. Accessed May 12, 2016. https://www.calstate.edu/acadsen/Records/Resolutions/2015-2016/documents/3230_Attachment_1.pdf

California State University Math Success. 2016. “Quantitative Reasoning Course.” Accessed May 12, 2016. http://www.csusuccess.org/quant_reason\
Common Core State Standard Initiative. 2016. “Standards for Mathematical Practice.” Accessed May 12, 2016. http://www.corestandards.org/Math/Practice/

Daro, P. 2008. “Mathematics for Whom? The Top of High School Meets the Bottom of College.” *The Noyce Foundation*. Carnegie Corporation of New York-Institute for Advanced Study Commission on Mathematics and Science Education. Accessed May 12, 2016. http://www.learningworksca.org/wp-content/uploads/2012/02/025-Phil-Daro-Mathematics-for-Whom.pdf

Ganter, S. & Barker W. 2004. “A Collective Vision: Voices of the Partner Disciples. The Curriculum Foundation Project.” *The Mathematical Association of America*. Accessed May 12, 2016. http://www.maa.org/sites/default/files/pdf/CUPM/crafty/curriculum-foundations.pdf

Ganter, S. & Harver, W. 2011. “Partner Discipline Recommendations for Introductory College Mathematics and the Implications for College Algebra.” *The Mathematical Association of America*. Accessed May 12, 2016. https://www.maa.org/sites/default/files/pdf/CUPM/crafty/introreport.pdf

Intersegmental Committee of the Academic Senates of the California Community Colleges, the California State University, and the University of California. “Statement on Competencies in Mathematics Expected of Entering College Students.” 2010. “Accessed May 12, 2016. http://icas-ca.org/Websites/icasca/images/ICAS-Statement-Math-Competencies-2013.pdf

Lumina Foundation. 2016. “The Degree Qualification Profile.” Accessed May 12, 2016. https://www.luminafoundation.org/files/resources/dqp.pdf

Microsoft. 2016. “A National Talent Strategy: Ideas for Securing Competitiveness and Economic Growth.” Accessed May 16, 2016. https://news.microsoft.com/download/presskits/citizenship/MSNTS.pdf

Newsom, Lt. Gov. Gavin. 2015. “Board of Admissions and Relations with Schools Computer Science Proposal.” Letter. Accessed May 4, 2016. http://www.sfchronicle.com/file/135/0/1350-BOARS%20Letter%20-%20Computer%20Science.pdf

Sacramento County Office of Education. 2015. “Executive Summary: Mathematics Framework for California Public School. Kinder Garden Through Grade Twelve.” Accessed May 12, 2016. https://www.scoe.net/castandards/Documents/summary_math_framework.pdf

Saxe, K., & Braddy, L. 2015. “A Common Vision for Undergraduate Mathematical Science Programs in 2025. The Mathematical Association of America (Incorporated).” Accessed May 12, 2016. http://www.maa.org/sites/default/files/pdf/CommonVisionFinal.pdf

Steen, L. A. 2001. “Mathematics and Democracy: The Case for Quantitative Literacy.” Accessed May 12, 2016. http://www.maa.org/sites/default/files/pdf/QL/MathAndDemocracy.pdf

The American Diplomat Project. 2004. “Ready or Not: Creating a High School Diploma that Counts.” Accessed May 4, 2016. http://www.achieve.org/files/ADPsummary_5.pdf

The American Diplomat Project. 2016. “Ready or No: Creating a High School Diploma that Counts.” Accessed May 4, 2016. http://www.achieve.org/files/ReadyorNot.pdf

The California State University Office of the Chancellor. 2008. “ELM Problem Book: Numbers and Data Algebra Geometry.” Accessed May 12, 2016. https://www.calstate.edu/SAS/ELMProblemBook01_02.pdf

The California State University Office of the Chancellor. 2014. “Focus on Mathematics Entry Level Mathematics Examination.” Accessed May 12, 2016. https://www.calstate.edu/sas/documents/focusonmath.pdf

The California State University Office of the Chancellor. 2011. “Practice Entry Level Mathematics Test.” Accessed May 12, 2016. https://www.csumathsuccess.org/uploads/02/cd/02cd8fc439f55d013e5aed752fc8868f/ELM-Practice-Test.pdf

E. Additional Rationale for 12th Grade Quantitative Reasoning

Not only is a fourth year of high school mathematics already recommended for all high school students intending to enroll in the CSU, but those students who are determined to be "Conditionally Ready" for college-level mathematics coursework are provided with an additional incentive to continue taking mathematic in their senior year of high school: By taking an approved senior-year math course and earning a grade of "C" or better, they do not need to participate in the Early Start summer program, nor will they need to take remedial mathematics courses at the CSU.

Students who take more mathematics in high school are less likely to need mathematics remediation. The College Board College-Bound Seniors Total Group Profile Reports[18] [19] [20] show, year after year, that the average SAT-Math score is less than 470 (33rd percentile)[21] for students who have only taken 3 years of high school, almost 520 (median) for students who have taken 4 years of high school mathematics, and over 570 (66th percentile) for students who have taken more than 4 years of high school mathematics. (For reference, the SAT score that the CSU accepts as indicating incoming proficiency in mathematics is 550.) ACT reports similar data[22] with the percentage of students reaching the proficiency level (which ACT defines as a 22 on the ACT-Math test; note that the CSU threshold is a score of 23) more than doubled (from 16% to 38%) as the years of high school mathematics increased from 3 to 3.5, and increased almost fourfold (from 16% to 62%) as the years of high school mathematics increased from 3 to 4.

Students who take higher level math classes in high school are less likely to take a remedial mathematics course in college, one-third less likely according to ACT[23] if they have taken any advanced mathematics course after Algebra II. The Utah System of Higher Education reports that students who successfully completed a course beyond Algebra II were more than twice as likely to successfully meet the Quantitative Literacy requirement in college.[24]

Finally, the Quantitative Reasoning Task Force surveyed a number of public universities and university systems across the United States and found such requirements to be in existence in at least 21 states. The related links were accessed on June 16, 2016. As not every university was checked, there may be additional institutions with this same requirement that do not appear on the following list:

Public Universities and Systems requiring 4 years of High School Mathematics

1. **Arizona**
   - Arizona State University[25]
   - Northern Arizona University[26]

[18] https://secure-media.collegeboard.org/digitalServices/pdf/sat/total-group-2015.pdf
[19] https://secure-media.collegeboard.org/digitalServices/pdf/sat/TotalGroup-2014.pdf
[20] http://media.collegeboard.com/digitalServices/pdf/research/2013/TotalGroup-2013.pdf
[21] http://wsac.wa.gov/sites/default/files/2014.CADRS.Overview.pdf
[22] http://files.eric.ed.gov/fulltext/ED496670.pdf
[23] http://files.eric.ed.gov/fulltext/ED496670.pdf
[24] http://higheredutah.org/high-school-math-critical-to-completion/
[25] https://students.asu.edu/freshman/requirements
[26] http://nau.edu/Admissions/Getting-Started/Requirements/Courses/

- University of Arizona[27]

2. **Arkansas**
   - Arkansas State University[28]
   - University of Arkansas (Fayetteville)[29]
   - University of Central Arkansas[30] [31]
3. **Colorado**
   - All four-year public institutions[32] [33] [34] [35]
4. **Florida**
   - State University System of Florida[36] [37]
5. **Georgia**
   - University System of Georgia[38]
6. **Indiana**
   - Purdue University System[39] [40]
7. **Louisiana**
   - Louisiana State University and A&M College (Baton Rouge) [41]
   - Southern University (Baton Rouge)[42]
   - University of New Orleans[43]
8. **Maryland**
   - University System of Maryland[44]
     - Note: The Maryland State Department of Education requires students, beginning with the entering 9th grade class in Fall 2014, to enroll in a mathematics course every year in high school as a high school graduation requirement.[45]
9. **Massachusetts**

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[27] http://admissions.arizona.edu/freshmen/entrance-requirements-and-guidelines
[28] http://www.astate.edu/info/admissions/undergraduate/hs-core-curriculum/index.dot
[29] http://admissions.uark.edu/apply/prepcore.php
[30] http://uca.edu/admissions/apply/freshman/
[31] http://www.arkansased.gov/public/userfiles/Learning_Services/Curriculum%20and%20Instruction/Smartcore%20Core/smartcore_course_2015_05142015.pdf
[32] http://highered.colorado.gov/Academics/Admissions/coursecompletion.html
[33] http://highered.colorado.gov/Publications/Policies/Current/i-partf2019_Revise.pdf
[34] http://www.colorado.edu/catalog/2015-16/content/minimum-academic-preparation-standards-maps
[35] http://admissions.colostate.edu/18units/
[36] http://www.flbog.edu/documents_regulations/regulations/6%20002%20FTIC%20Admissions%202_FINAL.pdf
[37] http://www.admissions.ufl.edu/ugrad/frqualify.html
[38] http://www.usg.edu/student_affairs/documents/Staying_on_Course.pdf
[39] http://admissions.purdue.edu/apply/highschoolcourses.php
[40] http://admissions.purdue.edu/apply/mathcourses.php
[41] http://sites01.lsu.edu/wp/admissions/become-a-tiger-2/freshmen/freshman-admission-requirements/
[42] http://www.subr.edu/index.cfm/page/325/n/1524
[43] http://www.uno.edu/admissions/freshman/academic-core-curriculum.aspx
[44] http://www.usmd.edu/newsroom/news/1021
[45] http://www.marylandpublicschools.org/MSDE/testing/hsg_qa/docs/MDHighSchoolGraduationRequirements.pdf

- Massachusetts State University System and University of Massachusetts System[46] [47] [48] [49] [50]
  - Note: The systemwide requirements take effect for students seeking admission in Fall 2016.
  - Note: University of Massachusetts Amherst specifically requires students to take mathematics in the senior year.

10. Minnesota

- University of Minnesota System[51]
  - Note: This requirement took effect for students seeking admission in Fall 2015.

11. Missouri

- University of Missouri System[52] [53]

12. Nebraska

- University of Nebraska-Lincoln[54]

13. New Mexico

- University of New Mexico[55]
- New Mexico State University[56]

14. North Carolina

- University of North Carolina System[57][58]

15. South Carolina

- All public senior colleges and universities colleges[59] [60] [61] [62]

16. Tennessee

- University of Tennessee at Chattanooga[63]
- University of Tennessee at Knoxville[64]
- University of Tennessee at Martin[65]
  - Note: The Tennessee Department of Education requires high schools students to earn four credits and to be enrolled in a mathematics course each year.[66]

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[46] http://www.mass.edu/shared/documents/admissions/admissionsstandards.pdf

[47] http://www.bridgew.edu/admissions/undergraduate/apply

[48] http://www.umass.edu/admissions/apply/admissions-requirements/freshman-admissions-requirements

[49] http://www.umassd.edu/undergraduate/about/

[50] https://www.uml.edu/admissions/freshmen-applicants.aspx

[51] http://admissions.tc.umn.edu/counselors/math_requirement.html

[52] https://www.umsystem.edu/ums/news/news_releases/um_enhances_admissions_policy

[53] http://admissions.missouri.edu/apply/freshmen/requirements/high-school-coursework.php

[54] https://admissions.unl.edu/apply.aspx#admission-requirements/freshmen

[55] http://admissions.unm.edu/future_students/admission-requirements.html

[56] http://admissions.nmsu.edu/files/2015/11/2016-NMSU-Undergraduate-Viewbook.pdf

[57] https://www.northcarolina.edu/prospective-students/minimum-admission-requirements

[58] http://admissions.unc.edu/minimum-course-requirements/

[59] http://www.che.sc.gov/Portals/0/CHE_Docs/publications/AnnualReports/Admissions_Standards_for_First-Time_Entering_Freshmen_%20FY2013-14.pdf

[60] http://www.che.sc.gov/CHE_Docs/AcademicAffairs/CollegePrepCourse_Prereqs101106.pdf

[61] http://www.sc.edu/about/offices_and_divisions/undergraduate_admissions/requirements/for_freshmen/required_high_school_courses/index.php

[62] http://www.scsu.edu/admissions/entrancerequirements/newfreshman.aspx

[63] http://www.utc.edu/admissions/apply/freshmanrequirements.php

[64] http://admissions.utk.edu/apply/requirements/

[65] http://www.utm.edu/departments/admissions/freshman.php

[66] http://www.tn.gov/education/topic/graduation-requirements

17. Texas
- The University of Texas at Austin[67]
- Texas A&M University (College Station)[68]

18. Virginia
- University of Virginia[69]

19. West Virginia
- University of West Virginia[70]

20. Wisconsin
- University of Wisconsin-Madison[71]

21. Wyoming
- University of Wyoming[72]

Additionally, some surveyed institutions, such as Indiana University Bloomington require 3.5 years of high school mathematics.[73] Others, such as Washington State University, require students to take a math-based quantitative course in their senior year of high school.[74] [75]

[67] https://admissions.utexas.edu/explore/prerequisites/general-requirements
[68] http://admissions.tamu.edu/freshman/coursework
[69] https://admission.virginia.edu/admission
[70] http://admissions.wvu.edu/how-to-apply/first-time-freshmen#anchor-freshmanreqs
[71] https://www.admissions.wisc.edu/apply/freshman/requirements.php
[72] http://www.uwyo.edu/admissions/freshman/admissions-requirements.html
[73] https://admissions.indiana.edu/apply/freshman/step-one.html
[74] http://catalog.wsu.edu/General/AcademicRegulations/Search/both/admission
[75] http://wsac.wa.gov/sites/default/files/2014.CADRS.Overview.pdf

F. Academic Senate CSU Resolution 3253-16

CALL FOR A CENTER FOR ADVANCEMENT OF INSTRUCTION IN MATHEMATICS

RESOLVED: That the Academic Senate of the California State University (ASCSU) encourage the establishment of a center to support mathematics instruction, analogous to the CSU Center for the Advancement of Reading (CAR); and be it further

RESOLVED: That the center have among its responsibilities:

a. Development of a fourth-year high school mathematics course, analogous to the Expository Reading and Writing Course (ERWC);
b. Professional development for, and evaluation of, the fourth-year mathematics course;
c. Professional development in effective mathematics/quantitative reasoning instruction; and
d. Policy alignment in matters affecting mathematics curriculum and instruction

; and be it further

RESOLVED: That the ASCSU distribute this resolution to the CSU Board of Trustees, CSU Chancellor, CSU campus Presidents, CSU campus Senate Chairs, CSU Provosts/Vice Presidents of Academic Affairs, CSU Math Council, CSU Deans of Colleges of Education, and the CSU Quantitative Reasoning Task Force.

***RATIONALE**: Currently, 27% of incoming CSU students arrive unprepared to succeed in college-level mathematics. In March 2016, the ASCSU passed AS-3244-16/APEP (Rev), "Support for Requiring a Fourth Year of Mathematics/Quantitative Reasoning for Admission to the California State University". Like the Center for the Advancement of Reading (CAR), this proposed center will provide leadership, support, training, and curricular resources in mathematics instruction for CSU faculty and California's K-12 teachers.*

Approved Unanimously – May 19-20, 2016