#10 Std. dev. of $\bar{x}$ values, sampled from a pop. of values, x, with mean μ, & std. dev. σ, with sample size n.

$$\sigma_{\bar{x}}^2 = Var(\bar{x}) = Var\left(\frac{\Sigma x}{n}\right) = \left(\frac{1}{n}\right)^2 Var(\overbrace{\Sigma x}^{n \text{ terms}})$$

$$= \frac{1}{n^2}(\underbrace{Var(x) + Var(x) + \cdots + Var(x)}_{n \text{ terms}})$$

$$= \frac{1}{n^2} \cdot n \cdot Var(x)$$

$$= \frac{1}{n} \cdot \sigma^2 = \frac{\sigma^2}{n}.$$

$$\sigma_{\bar{x}} = \sqrt{Var(\bar{x})} = \sqrt{\frac{\sigma^2}{n}} = \frac{\sigma}{\sqrt{n}}.$$

$$\left(z = \frac{\bar{x} - \mu}{\sigma/\sqrt{n}} \qquad E = z_{\alpha/2} \cdot \frac{\sigma}{\sqrt{n}} \right)$$

I could also ask you to prove $\mu_{\bar{x}} = \mu$.

#11 Std. dev. of $\hat{p}$ values - start with variance (p = pop. proportion, $\hat{p} = \frac{x}{n}$ = sample proportion)

$$\sigma_{\hat{p}}^2 = Var(\hat{p})$$

$$= Var\left(\frac{x}{n}\right) = \frac{1}{n^2} Var(x) = \frac{1}{n^2} npq = \frac{pq}{n}$$

$$\sigma_{\hat{p}} = \sqrt{Var(\hat{p})} = \sqrt{\frac{pq}{n}}.$$

$$\left(z = \frac{\hat{p} - p}{\sqrt{\frac{pq}{n}}}, \quad E = z_{\alpha/2} \cdot \sqrt{\frac{pq}{n}} \right)$$

I could ask you to prove $\mu_{\hat{p}} = p$.

(In a binomial experiment
$\mu = E(x) = np$
$\sigma = \sqrt{npq}$, $Var(x) = \sigma^2 = npq$)