

# One-Way Analysis of Variance

ANOVA

## Section 12.1



Equal sample sizes for each sample.

Comparing means from 3 or more samples.\*

\* This is simplest when all sample sizes ( $n$ ) are all equal.

Temperatures of sick patients after receiving a treatment

|             | Placebo | Aspirin | Anacin | Tylenol | Bufferin |                                              |
|-------------|---------|---------|--------|---------|----------|----------------------------------------------|
|             | 98.3    | 96.8    | 97.2   | 95.1    | 96.1     | $H_0: \mu_1 = \mu_2 = \mu_3 = \mu_4 = \mu_5$ |
|             | 98.7    | 96.6    | 97.4   | 95.7    | 96.5     |                                              |
|             | 98.1    | 95.3    | 96.9   | 94.4    | 95.4     |                                              |
|             | 99.9    | 98.1    | 97.8   | 97.2    | 98.2     |                                              |
|             | 96.2    | 94.6    | 94.7   | 93.7    | 94.5     |                                              |
|             | 98.6    | 96.6    | 97.0   | 95.7    | 96.5     |                                              |
|             | 98.3    | 96.2    | 96.8   | 95.3    | 96.2     | $H_1: \text{at least one mean is different}$ |
| → $\bar{x}$ | 98.30   | 96.20   | 96.83  | 95.30   | 96.20    |                                              |
| $s$         | 1.10    | 1.11    | 1.00   | 1.11    | 1.13     |                                              |
| $s^2$       | 1.21    | 1.22    | 1.00   | 1.22    | 1.29     |                                              |
| $n$         | 7       | 7       | 7      | 7       | 7        | $k = 5$                                      |

Hypotheses:  $H_0: \mu_1 = \mu_2 = \mu_3 = \dots = \mu_k$ ,  $H_1: \text{at least one mean differs}$  ↖  $k$  populations/treatments  
 Gather data:  $\bar{x}$ ,  $s^2$ ,  $n$  from each sample (here we assume all  $n$ 's are equal)  
 ✓  $n = \text{common sample size}$ ,  $k = \# \text{ of pop's/treatments}$   
 Requirements: All populations are normal with equal variances (these are not strict).

Test Statistic:  $F = \frac{n S_{\bar{x}}^2}{S_p^2}$   $S_{\bar{x}}^2 = (\text{std. dev. of } \bar{x} \text{ val's})^2$   
 $S_p^2 = \text{mean of } s^2 \text{ values.}$

$$S_{\bar{x}}^2 = 1.1120^2 \approx 1.2366, \quad S_p^2 = \frac{1.21 + 1.22 + 1.00 + 1.22 + 1.29}{5} = 1.188$$

$$\text{T.S. } F = \frac{n S_{\bar{x}}^2}{S_p^2} = \frac{7 \cdot 1.2366}{1.188} \approx 7.2864$$