

A new statistic in a binomial experiment:

Sample Proportion $\rightarrow \hat{p} = \frac{x}{n}$ $\leftarrow$ # of successes
 $\leftarrow$ # of trials

This is different from p :

p = population = probability of success,

$\hat{p}$ = the statistic that estimates the parameter p .

Each x corresponds to a $\hat{p} = \frac{x}{n}$ so they have the same probabilities. They are both binomial.

Review for Binomial Exper:
 $\mu = E(x) = np$
 $\sigma^2 = \text{Var}(x) = npq$

But binomial probabilities are appx. normal if $np \geq 10$ & $nq \geq 10$.

$$E(\hat{p}) = E\left(\frac{x}{n}\right) = E\left(\frac{1}{n} \cdot x\right) = \frac{1}{n} (E(x)) = \frac{1}{n} \cdot np = p$$

So $\hat{p}$ is an unbiased estimator of p .

Also

$$\begin{aligned} \text{Var}(\hat{p}) &= \text{Var}\left(\frac{x}{n}\right) = \text{Var}\left(\frac{1}{n} \cdot x\right) = \frac{1}{n^2} \text{Var}(x) \\ &= \frac{1}{n^2} \cdot npq \end{aligned}$$

So the standard deviation is the square root of variance

$$= \frac{pq}{n}$$

$$\sigma_{\hat{p}} = \sqrt{\frac{pq}{n}} = \text{standard error of } \hat{p} \text{ values.}$$