

Properties of Expected Value & Variance

$$1) E(x-y) = E(x) - E(y)$$

$$2) \text{Var}(x-y) = \text{Var}(x) + \text{Var}(y)$$

$$3) E(x+y) = E(x) + E(y)$$

$$4) \text{Var}(x+y) = \text{Var}(x) + \text{Var}(y)$$

$$5) E(kx) = k \cdot E(x)$$

$$6) \text{Var}(kx) = k^2 \cdot \text{Var}(x)$$

7) If x & y have a bell-shaped/normal distribution, then $x-y$, $x+y$, kx ($k \neq 0$) are all bell-shaped.

$$x \sim \{5, 7, 7, 9\} \rightarrow 2x: \{10, 14, 14, 18\}$$

$$E(2x) = 14 = 2 \cdot 7 = 2 \cdot E(x)$$

$$\text{Var}(2x) = 8$$

$$= 4 \cdot 2$$

$$= 4 \cdot \text{Var}(x)$$

$$= 2^2 \cdot \text{Var}(x)$$

5.2#5

$$\begin{cases} \mu_x = E(x) = 7, & \mu_y = E(y) = 4 \\ \sigma_x = 2, & \sigma_y = 3 \\ \text{Var}(x) = 4, & \text{Var}(y) = 9 \end{cases}$$

$$\begin{aligned} \mu_{3x-2y} &= E(3x-2y) = E(3x) - E(2y) = 3E(x) - 2E(y) \\ &= 3 \cdot 7 - 2 \cdot 4 = 21 - 8 = 13 \end{aligned}$$

$$\begin{aligned} \text{Var}(3x-2y) &= \text{Var}(3x) + \text{Var}(2y) \\ &= 3^2 \cdot \text{Var}(x) + 2^2 \cdot \text{Var}(y) \\ &= 9 \cdot 4 + 4 \cdot 9 = 72 \end{aligned}$$

$$\sigma_{3x-2y} = \sqrt{\text{Var}(3x-2y)} = \sqrt{72} = 8.485$$