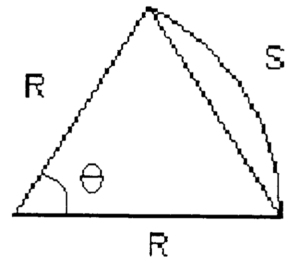
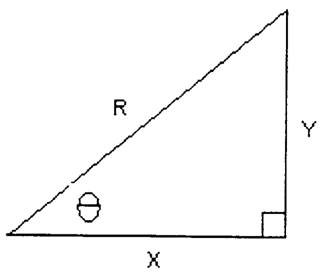
Trigonometry: Properties, Special Angles & Identities

The angle θ is defined by $\theta = \frac{s}{r}$, where s is the distance along the arc, and r is the distance from the center to the arc. This number is in radians. To convert to degrees, multiply by $\frac{180}{\pi}$.





The six trigonometric identities are all ratios of the three sides of the triangle. They are all definitions and should be memorized

Sine of θ = $\sin\theta = \frac{y}{R}$

Cosine of θ = $\cos\theta = \frac{x}{R}$

Tangent of θ = $\tan\theta = \frac{y}{x}$

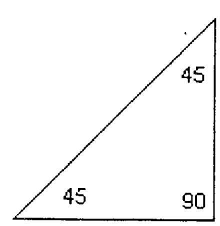
cosecant of θ = $\csc\theta = \frac{R}{y} = \frac{1}{\sin\theta}$

secant of θ = $\sec\theta = \frac{R}{X} = \frac{1}{\cos\theta}$

cotangent of θ = $\frac{X}{Y} = \frac{1}{\tan\theta}$

There are some triangles from geometry with special angles, and the values of the six trig. Functions for these angles should be memorized. The first triangle is the 45°-45°-90°.

Sin 45° = $\frac{\sqrt{2}}{2}$	csc 45° = $\frac{2}{\sqrt{2}} = \frac{2\sqrt{2}}{2} = \sqrt{2}$
Cos 45° = $\frac{\sqrt{2}}{2}$	sec 45° = $\frac{2}{\sqrt{2}} = \frac{2\sqrt{2}}{2} = \sqrt{2}$
Tan 45° = $\frac{\sqrt{2}}{\sqrt{2}}$ =1	cot 45° = $\frac{\sqrt{2}}{\sqrt{2}}$ =1



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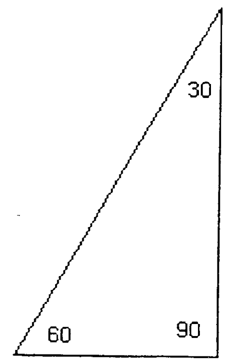
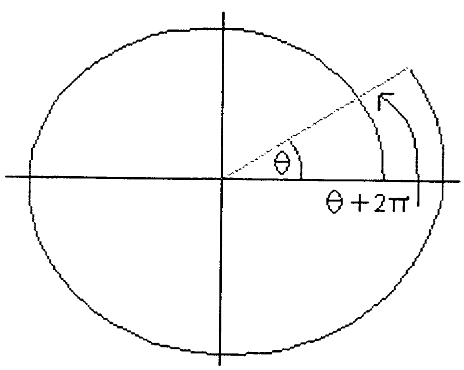
The other triangle is the 30°-60°-90°

$\sin 60° = \frac{\sqrt{3}}{2}$	$\csc 60° = \frac{2}{\sqrt{3}} = \frac{2\sqrt{3}}{3}$
$\cos 60° = \frac{1}{2}$	$\sec 60° = \frac{2}{1}$
$\tan 60° = \frac{\sqrt{3}}{1} = \sqrt{3}$	$\cot 60° = \frac{1}{\sqrt{3}} = \frac{\sqrt{3}}{3}$
$\sin 30° = \frac{1}{2}$	$\csc 30° = \frac{2}{1} = 2$
$\cos 30° = \frac{\sqrt{3}}{2}$	$\sec 30° = \frac{2}{\sqrt{3}} = \frac{2\sqrt{3}}{3}$
$\tan 30° = \frac{1}{\sqrt{3}} = \frac{\sqrt{3}}{3}$	$\cot 30° = \frac{\sqrt{3}}{1} = \sqrt{3}$



Suppose we wanted to find an angle other than the ones we have memorized. There are tools in trigonometry that allow us to do this. The first is the idea of coterminal angles. Because of the way the unit circle works, any angle can be represented the exact same way by adding or subtracting 360° (or 2π in radians). Here is a graph to help see why.



The first angle is the small one, represented by θ. The larger angle goes all the way around and comes back to the exact same angle, as the straight line shows. Since a circle is 2π radians, if we add a complete circle to the angle, we arrive right back where we started from.

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The second set of these tools is the angle addition formulas. These formulas allow us to find the sin, cosine, or tangent of the *sum* of two angles. The formulas for sine, cosine, and tangent are:

- $\sin(A+B) = \sin(A)*\cos(B) + \sin(B)*\cos(A)$
- $\cos(A+B) = \cos(A)\cos(B) - \sin(A)\sin(B)$
- $\tan(A+B) = \dfrac{\tan(A) + \tan(B)}{1 - \tan(A)\tan(B)}$

Angle subtraction formulas can be obtained by substituting (-B) for (B) and evaluating the formulas. As an example,

$$\cos(75°) = \cos(30°+45°)$$

$$= \cos(30°)\cos(45°) - \sin(30°)\sin(45°)$$

$$=\left(\frac{\sqrt{3}}{2}\right)\left(\frac{\sqrt{2}}{2}\right)-\left(\frac{1}{2}\right)\left(\frac{\sqrt{2}}{2}\right)=\frac{\sqrt{6}-\sqrt{2}}{4}$$

Almost all of the other trig identities can be derived from clever substitutions into these three basic formulas. For example, the double angle identities can be obtained by setting B equal to A.

- $\sin(2A) = \sin(A+A) = \sin(A)\cos(A)+\sin(A)\cos(A) = 2\sin(A)\cos(A)$

Similarily,

- $\cos(2A) = \cos^2(A) - \sin^2(A)$, and by the Pythagorean theorem,
- $\cos(2A) = 2\cos^2(A) - 1 = 1- 2\sin^2(A)$
- $\tan(2A) = \dfrac{2\tan(A)}{1 - \tan^2(A)}$

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These new formulas are new tools to work with in making new formulas. They can be used to derive the half-angle formulas.

If $\cos(2A) = 1-2\sin^2(A)$, then $2\sin^2(A) = 1-\cos(2A)$, and so

- $\sin(A) = \sqrt{\dfrac{1-\cos(2A)}{2}}$.

This becomes particularly useful when we make a small substitution in the angles. If we let $B=2A$, then $A=\dfrac{B}{2}$, and so

- $\sin(\dfrac{B}{2}) = \sqrt{\dfrac{1-\cos(B)}{2}}$.

Similar work will give us

- $\cos(\dfrac{B}{2}) = \sqrt{\dfrac{1+\cos B}{2}}$.
- $\tan(\dfrac{B}{2}) = \pm\sqrt{\dfrac{1-\cos B}{1+\cos B}} = \dfrac{\sin B}{1+\cos B} = \dfrac{1-\cos B}{\sin B}$

Clever combinations and substitutions of these and other trigonometric identities will yield even more identities, such as the sum-to-product and product-to-sum identities. For a more complete list of trig identities, see our other handout available on the back wall or ask a tutor.