Trigonometric Identities

Here is a list of many identities from trigonometry. These identities may be used to verify or establish other identities.

Reciprocal Identities:

$$\cot\theta = \frac{1}{\tan\theta} \qquad \sec\theta = \frac{1}{\cos\theta} \qquad \csc\theta = \frac{1}{\sin\theta}$$

Ratio Identities:

$$\tan\theta = \frac{\sin\theta}{\cos\theta} \qquad \cot\theta = \frac{\cos\theta}{\sin\theta}$$

Negative-Angle Identities:

$$\sin(-\theta) = -\sin\theta \qquad \cos(-\theta) = \cos\theta \qquad \tan(-\theta) = -\tan\theta$$

$$\csc(-\theta) = -\csc\theta \qquad \sec(-\theta) = \sec\theta \qquad \cot(-\theta) = -\cot\theta$$

Co-function Identities:

$$\sin(90° - \theta) = \cos\theta \qquad \cos(90° - \theta) = \sin\theta$$

$$\tan(90° - \theta) = \cot\theta \qquad \cot(90° - \theta) = \tan\theta$$

$$\sec(90° - \theta) = \csc\theta \qquad \csc(90° - \theta) = \sec\theta$$

Pythagorean Identities:

$$\sin^2\theta + \cos^2\theta = 1$$

$$\tan^2\theta + 1 = \sec^2\theta$$

$$1 + \cot^2\theta = \csc^2\theta$$

Sum & Difference Identities:

$$\cos(\alpha + \beta) = \cos\alpha\cos\beta - \sin\alpha\sin\beta \qquad \cos(\alpha - \beta) = \cos\alpha\cos\beta + \sin\alpha\sin\beta$$

$$\sin(\alpha + \beta) = \sin\alpha\cos\beta + \cos\alpha\sin\beta \qquad \sin(\alpha - \beta) = \sin\alpha\cos\beta - \cos\alpha\sin\beta$$

$$\tan(\alpha + \beta) = \frac{\tan\alpha + \tan\beta}{1 - \tan\alpha\tan\beta} \qquad \tan(\alpha - \beta) = \frac{\tan\alpha - \tan\beta}{1 + \tan\alpha\tan\beta}$$

Double-Angle Identities:

$$\sin 2\alpha = 2\sin\alpha\cos\alpha$$

$$\cos 2\alpha = \cos^2\alpha - \sin^2\alpha = 2\cos^2\alpha - 1 = 1 - 2\sin^2\alpha$$

$$\tan 2\alpha = \frac{2\tan\alpha}{1 - \tan^2\alpha}$$

Half-Angle Identities:

$$\sin\frac{\alpha}{2} = \pm\sqrt{\frac{1 - \cos\alpha}{2}}$$

$$\cos\frac{\alpha}{2} = \pm\sqrt{\frac{1 + \cos\alpha}{2}}$$

$$\tan\frac{\alpha}{2} = \pm\sqrt{\frac{1 - \cos\alpha}{1 + \cos\alpha}} = \frac{\sin\alpha}{1 + \cos\alpha} = \frac{1 - \cos\alpha}{\sin\alpha}$$

Product to Sum Identities:

$$\sin\alpha\cos\beta = \frac{\sin(\alpha + \beta) + \sin(\alpha - \beta)}{2}$$

$$\sin\alpha\sin\beta = \frac{\cos(\alpha - \beta) - \cos(\alpha + \beta)}{2}$$

$$\cos\alpha\cos\beta = \frac{\cos(\alpha + \beta) + \cos(\alpha - \beta)}{2}$$

Sum to Product Identities:

$$\sin\alpha + \sin\beta = 2\sin\left(\frac{\alpha + \beta}{2}\right)\cos\left(\frac{\alpha - \beta}{2}\right)$$

$$\sin\alpha - \sin\beta = 2\cos\left(\frac{\alpha + \beta}{2}\right)\sin\left(\frac{\alpha - \beta}{2}\right)$$

$$\cos\alpha + \cos\beta = 2\cos\left(\frac{\alpha + \beta}{2}\right)\cos\left(\frac{\alpha - \beta}{2}\right)$$

$$\cos\alpha - \cos\beta = -2\sin\left(\frac{\alpha + \beta}{2}\right)\sin\left(\frac{\alpha - \beta}{2}\right)$$

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The following is an example of how to use the given identities to verify another identity.

ex: Verify the following:

$$\frac{\sin 3x - \sin x}{\cos x - \cos 3x} = \cot(2x)$$

→ When we are asked to verify (prove) an identity we must work on each side of the equal sign independently. This means that we cannot use the addition or multiplication properties of equality for example, since these involve performing operations on both sides of the equal sign. In this example, we only work on the left side of the equal sign, but this is not the only verification possible for this problem.

$$\frac{2\cos\left(\frac{3x+x}{2}\right)\sin\left(\frac{3x-x}{2}\right)}{-2\sin\left(\frac{x+3x}{2}\right)\sin\left(\frac{x-3x}{2}\right)} =$$

→ We rewrite our differences in our numerator and denominator as products using the sum to product identities.

$$\frac{2\cos(2x)\sin(x)}{-2\sin(2x)\sin(-x)} =$$

→ Simplifying inside our parentheses gives us this expression.

$$\frac{2\cos(2x)\sin(x)}{2\sin(2x)\sin(x)} =$$

→ Since sin (–x) = –sin (x), we can rewrite like this.

$$\frac{\cos(2x)}{\sin(2x)} =$$

→ After canceling, we end up here.

$$\cot(2x) = \cot(2x)$$

→ The last step is a ratio identity. When we manage to make one side of the equal sign match the other side, we are done with the verification.

Note: *Verification of identities is sometimes a slippery endeavor, but here are some tips. If you don't know how to start, you might try to make each trigonometric function into a sine or cosine. Sometimes this helps to make the identities you remember easier to recognize. Also, I find that it is often easier to begin on the more complicated side of the equal sign. Notice that in the above example, we started on the left side, which was more complicated than cot(2x) on the right. As you work, keep in mind what the goal is. For instance, in the above example, we needed to get an angle of "2x" somewhere on the left side, since this is the argument of the cotangent given on the right.*

Note Also: *Memorizing the identities given on the first page would obviously be a great benefit, but if you can't remember all of them it may still be possible to do the problem. For instance, in the above example, the verification may be accomplished without using the sum to product identities (these are the ones I often forget), but it is more time-consuming (trust me, I did it this way the first time).*

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