

Rationalize the denominator and multiply with Radicals

Rationalizing is done to remove the radical from the denominator of a fraction. We will consider three cases involving square roots. It will be helpful to remember how to reduce a radical when continuing with these problems.

Note: Squaring a radical will eliminate the radical. Because $\sqrt{x} \cdot \sqrt{x} = (\sqrt{x})^2 = x$ \Leftarrow this equals x

Case 1: Single radical in the denominator

- $\frac{2y}{\sqrt{18y^3}}$ \Leftarrow So we can eliminate the radical from the denominator by doubling it.
- $\frac{2y}{\sqrt{18y^3}} \cdot \frac{\sqrt{18y^3}}{\sqrt{18y^3}}$ \Leftarrow But anything you do the bottom of a fraction you must do to the top.
- $\frac{2y}{\sqrt{18y^3}} \cdot \frac{\sqrt{18y^3}}{\sqrt{18y^3}}$ \Leftarrow Now multiply across $\Rightarrow \frac{2y\sqrt{18y^3}}{(\sqrt{18y^3})^2}$ \Leftarrow then simplify both top and bottom.
- $\frac{2y\sqrt{9 \cdot 2y^2y}}{18y^3}$ \Leftarrow Reduce the radical on top $\Rightarrow \frac{2 \cdot 3 \cdot y \cdot y\sqrt{2y}}{18y^3}$
- $\frac{6y^2\sqrt{2y}}{18y^3}$ \Leftarrow Reduce the fraction using the terms outside the radical $\Rightarrow \frac{\sqrt{2y}}{3y}$ Done!

Case 2: Entire fraction is in the radical

- $\sqrt{\frac{50x^3}{6x^4}}$ \Leftarrow First, reduce under the radical whenever possible.
- $\sqrt{\frac{25}{3x}}$ \Leftarrow Then, split the radical top and bottom $\Rightarrow \frac{\sqrt{25}}{\sqrt{3x}}$ Continue as in **case 1** above.
- $\frac{\sqrt{25}}{\sqrt{3x}} \cdot \frac{\sqrt{3x}}{\sqrt{3x}} \Rightarrow \frac{\sqrt{75x}}{\sqrt{9x^2}}$ \Leftarrow Simplify the radicals $\Rightarrow \frac{5\sqrt{3x}}{3x}$ Done!

Rationalizing two terms in the denominator requires the conjugate.

Definition: Conjugates are the same two terms separated by different signs.

For instance: $\sqrt{5} - \sqrt{11}$ and $\sqrt{5} + \sqrt{11}$ are conjugates of each other.

Example: Rationalize the denominator

- $\frac{5}{\sqrt{2} - \sqrt{3}} \Rightarrow$ The conjugate of $\sqrt{2} - \sqrt{3}$ is $\sqrt{2} + \sqrt{3}$ so $\Rightarrow \frac{5}{\sqrt{2} - \sqrt{3}} \cdot \frac{\sqrt{2} + \sqrt{3}}{\sqrt{2} + \sqrt{3}} \Leftarrow$ Multiply
- $\frac{5(\sqrt{2} + \sqrt{3})}{(\sqrt{2} - \sqrt{3})(\sqrt{2} + \sqrt{3})} \Rightarrow$ FOIL the denominator $\Rightarrow \frac{5(\sqrt{2} + \sqrt{3})}{\sqrt{2}\sqrt{2} + \sqrt{2}\sqrt{3} - \sqrt{2}\sqrt{3} - \sqrt{3}\sqrt{3}}$
- $\frac{5(\sqrt{2} + \sqrt{3})}{\sqrt{4} + \sqrt{6} - \sqrt{6} - \sqrt{9}} \Rightarrow \frac{5(\sqrt{2} + \sqrt{3})}{2 - 3} \Rightarrow \frac{5(\sqrt{2} + \sqrt{3})}{-1}$
- Reduce bringing the minus sign to the numerator $\Rightarrow -5(\sqrt{2} + \sqrt{3})$ Done ☺

Multiply and FOIL with radicals.

Example 1: Distribution with a single term

- $\sqrt{5y}(\sqrt{5} - \sqrt{y}) \Leftarrow$ Use distribution $\Rightarrow \sqrt{5y} \cdot \sqrt{5} - \sqrt{5y} \cdot \sqrt{y}$

Rules for radicals $\Rightarrow \sqrt{5y} \cdot \sqrt{5} = \sqrt{25y}$ and $\sqrt{5y} \cdot \sqrt{y} = \sqrt{5y^2}$

- $\sqrt{25y} - \sqrt{5y^2} \Leftarrow$ Simplify the radicals $\Rightarrow 5\sqrt{y} - y\sqrt{5}$ Done!

Example 2: FOIL multiply two terms with two terms

- $(\sqrt{2x} - \sqrt{3y})(\sqrt{x} - \sqrt{3y}) \Rightarrow$

	First	Outer	Inner	Last
\Rightarrow	$\sqrt{2x}\sqrt{x}$	$-\sqrt{2x}\sqrt{3y}$	$-\sqrt{3y}\sqrt{x}$	$+\sqrt{3y}\sqrt{3y}$

- $\sqrt{2x^2} - \sqrt{6xy} - \sqrt{3xy} + \sqrt{9y^2}$ Answer $x\sqrt{2} - \sqrt{6xy} - \sqrt{3xy} + 3y$

Example 3: Two terms squared; double then FOIL

- $(3 - \sqrt{w})^2 \Rightarrow (3 - \sqrt{w})(3 - \sqrt{w}) \Leftarrow$ Must write two terms then FOIL

First Outer Inner Last

- $3 \cdot 3 - 3\sqrt{w} - 3\sqrt{w} + \sqrt{w}\sqrt{w} \Leftarrow$ combine radicals and simplify

- $9 - 6\sqrt{w} + w$ Finished because none of the terms are like terms.

Practice Problems

Rationalize and multiply the radicals.

$$1) \sqrt{\frac{24x^3}{8y}}$$

$$2) \frac{\sqrt{27}}{\sqrt{6}}$$

$$3) \frac{\sqrt{3}}{\sqrt{2y}}$$

$$4) \frac{2}{\sqrt{6}-3}$$

$$5) \frac{\sqrt{5}+2}{2-\sqrt{3}}$$

$$6) \sqrt{\frac{4}{x}}$$

$$7) \sqrt{7}(8+\sqrt{12})$$

$$8) (\sqrt{6}-\sqrt{3})(\sqrt{3}+\sqrt{18})$$

$$9) (\sqrt{3}-5)(\sqrt{3}+5)$$

$$10) (\sqrt{2x}-7)^2$$

Answer Key

$$1) \frac{x\sqrt{3xy}}{y}$$

$$2) \frac{3\sqrt{2}}{2}$$

$$3) \frac{\sqrt{6y}}{2y}$$

$$4) -\frac{2\sqrt{6}+6}{3}$$

$$5) 2\sqrt{5} + \sqrt{15} + 4 + 2\sqrt{3}$$

$$6) \frac{2\sqrt{x}}{x}$$

$$7) 8\sqrt{7} + 2\sqrt{21}$$

$$8) 6\sqrt{3} + 3\sqrt{2} - 3 - 3\sqrt{6}$$

$$9) -22$$

$$10) 2x - 14\sqrt{2x} + 49$$