Math 51
Worksheet

How to solve Equations with Radical Expressions

Checking your answer on is required because solutions may be extraneous.

I. Solve an equation with a single square root using the squaring property of equality.

Consider the example and try to come up with the solution. $\sqrt{x} = 7$? The solution is $x = 49$

Check: $\sqrt{49} = 7$ It works.

Another way to arrive at the answer is to square both sides of the equation. Remember, anything you do to one side of the equation you must do to the other side.

Example: $\sqrt{x} = 7 \quad \Rightarrow$ square both sides $\Rightarrow \quad \left(\sqrt{x}\right)^2 = (7)^2 \quad \Leftarrow$ The square and square root cancel out on the left.

So, $x = (7)^2 \quad \Rightarrow \quad x = 49$ Done!

Note: The radical must be isolated on one side of the equal sign before you can square both side.

Example: Solve $2\sqrt{x-5} + 3 = 21$ The radical is not isolated.

- $2\sqrt{x-5} + 3 - 3 = 21 - 3$ $\Leftarrow$ Subtract the three first from both sides.
- $\frac{2}{2}\sqrt{x-5} = \frac{18}{2}$ $\Leftarrow$ Divide two on both sides.
- $\sqrt{x-5} = 9$ $\Leftarrow$ Now we can square both sides.
- $\left(\sqrt{x-5}\right)^2 = (9)^2$ $\Leftarrow$ The square and the square root cancel out.
- $x - 5 = (9)^2$ $\Leftarrow$ Solve for x
- $x = 81 + 5 \quad \Rightarrow \quad x = 86$ $\Leftarrow$ Must check
- Check: $2\sqrt{86-5} + 3 = 21 \quad \Rightarrow \quad 2\sqrt{81} + 3 = 21$

$2(9) + 3 = 21 \quad \Rightarrow \quad 21 = 21$ **It works**! Answer: $\boxed{x = 86}$

II. The next two examples will involve **FOIL**ing.

- $\sqrt{51-x} = x + 5$ $\Leftarrow$ The radical is isolated so we square both sides.
- $\left(\sqrt{51-x}\right)^2 = (x+5)^2$ $\Leftarrow$ Cancel out the radical on the left and FOIL on the right.

- $51-x=(x+5)(x+5)$ $\Rightarrow$ $51-x=x^2+10x+25$ ← Set equal to zero and solve.
- $0=x^2+11x-26$ ⇐ Factor by **FOIL**ing
- $0=(x+13)(x-2)$ ⇐ Solve
- So, $x=-13$ *or* $x=2$ ⇐ Must check both.

Check: $\sqrt{51-(-13)}=-13+5 \Rightarrow 8\neq -8$ Check: $\sqrt{51-(2)}=(2)+5 \Rightarrow 7=7$

Only $x=2$ **Works!** Answer: $\boxed{x=2}$

III. This example will require you to square twice because there are two radicals in the problem.

- $\sqrt{3x}-\sqrt{2x-5}=-2$ ⇐ First, isolate one radical by adding $\sqrt{2x-5}$ to both sides.
- $\sqrt{3x}=\sqrt{2x-5}-2$ ⇐ One radical is isolate so we square both sides.
- $\left(\sqrt{3x}\right)^2=\left(\sqrt{2x-5}-2\right)^2$ ⇒ You must **FOIL** on the right ⇒ $3x=\left(\sqrt{2x-5}-2\right)\left(\sqrt{2x-5}-2\right)$
- $3x=\left(\sqrt{2x-5}\right)^2-2\sqrt{2x-5}-2\sqrt{2x-5}+4$ ⇐ Combine both radicals in the middle.
- $3x=2x-5-4\sqrt{2x-5}+4$ ⇒ One radical is left so it must be isolated ⇒ $3x=2x-1-4\sqrt{2x-5}$
- $x+1=-4\sqrt{2x-5}$ ⇐ This time we do not divide both sides by -4 because a fraction would be made on the left.
- $(x+1)^2=\left(-4\sqrt{2x-5}\right)^2$ ⇐ Once again square both sides
- $(x+1)(x+1)=16(2x-5)$ ⇐ **FOIL** on the left
- $x^2+2x+1=32x-80$ ⇐ Set to zero and solve
- $x^2-30x+81=0$ ⇒ Factor by **FOIL**ing ⇒ $(x-27)(x-3)=0$ ← Set each factor equal to zero.

So, $x=27$ *or* $x=3$ ⇐ Must check both.

Check: $\sqrt{3(3)}-\sqrt{2(3)-5}=-2 \Rightarrow 2\neq -2$ Check: $\sqrt{3(27)}-\sqrt{2(27)-5}=-2 \Rightarrow 2\neq -2$

Neither answer works, therefore the answer is no solution. Answer: $\emptyset$

Practice Problems

Perform the indicated operation and reduce all radicals.

1) $6-\sqrt{y-5}=3$

2) $\sqrt{2-x}+3=x+7$

3) $\sqrt{x}+4=0$

4) $\sqrt{4y+1}=y-1$

5) $\sqrt{2p+3}=\sqrt{5p-3}$

6) $\sqrt{5x+4}=3\sqrt{x}$

7) $2x=\sqrt{4x^2+6x-12}$

8) $\sqrt{2x}=\sqrt{x+7}-1$

9) $\sqrt{x-1}=4$

10) $\sqrt{x-2}+\sqrt{x-13}=11$

Answer Key

1) y = 14

2) x = -2

3) No solution

4) y = 6

5) p = 2

6) x = 1

7) x = 2

8) x = 2

9) x = 17

10) x = 38