Math 51
Worksheet

Introduction to Radicals

Radicals will be: *evaluated*, *simplified*, *added, subtracted, multiplied* and *divided.*

- Square root or radical notation $\Rightarrow$ $\sqrt[\text{index}]{\text{Radicand}}$

What number multiplied by itself gives the number under the radical?

- Perfect square numbers evaluate to rational numbers
  - $\sqrt{25} = 5$ Because $5 \cdot 5 = 25$
- Other numbers require the use of a calculator and evaluate to irrational number
  - $\sqrt{13} = 3.605551275...$
- Can you take the square root of a negative number? Why?
  - $\sqrt{-16} = ???$ Does $(-4)(-4) = -16$ No. So, the answer is not a real number.

Square roots have an index of 2 other radicals are possible index 3, 4, 5 and so on. But the definition is the same.

- $\sqrt[3]{125} = 5$ **Because**, $5 \cdot 5 \cdot 5 = 125$ or $\sqrt[4]{81} = 3$ **Because**, $3 \cdot 3 \cdot 3 \cdot 3 = 81$ and so on.

<u>Simplifying radicals</u>

What is the square root of a variable raised to a power? $\sqrt{x^6} = x^3$ Why? Because $x^3 \cdot x^3 = x^6$

Another way to think about it is divide the power by the index. $\sqrt{x^6} = x^{\frac{6}{2}} = x^3$ If the power does not divide evenly then the variable must be factored first so that the power can be divided by the index.

Example: $\sqrt[4]{x^{14}} = x^{\frac{14}{4}}$ The power does not reduce to a whole number. So, factor the part that does:

$\sqrt[4]{x^{12}x^2} \Rightarrow x^{\frac{12}{4}}\sqrt[4]{x^2} \Rightarrow x^3\sqrt[4]{x^2}$ $\Leftarrow$ The remainder stays inside the radical.

Next Example: Simplify

- $\sqrt{108x^5y^8}$ Problem? 108 is not a perfect square but one of its factors maybe.
- $\sqrt{(36)(3)x^5y^8}$ What about x^5 5 divide 2 does not work, so that must be factored as well.
- $\sqrt{(36)(3)x^4xy^8}$ Everything that can be simplified comes out of the radical and everything else must stay inside.
- $\sqrt{(36)(3)x^4xy^8} \Rightarrow 6x^{\frac{4}{2}}y^{\frac{8}{2}}\sqrt{3x} \Rightarrow 6x^2y^4\sqrt{3x}$

The same rules apply to all other radicals just the index changes.

Example: $\sqrt[3]{135x^7y^{15}} \Rightarrow \sqrt[3]{(27)(5)x^6xy^{15}} \Rightarrow 3x^{\frac{6}{3}}y^{\frac{15}{3}}\sqrt[3]{5x}$ Reduce all terms.

Answer: $3x^2y^5\sqrt[3]{5x}$

Multiply/Divide radicals: If the indexes are the same write as one radical.

Multiply

- $\sqrt[4]{2x^3y}\cdot\sqrt[4]{8x^5y^2}$ $\Leftarrow$ The indexes are the same "4". So, write as one radical first.
- $\sqrt[4]{2x^3y\cdot 8x^5y^2}$ $\Leftarrow$ Everything inside will multiply. Note: Remember your power rules.
- $\sqrt[4]{2\cdot 8x^3x^5\,y\,y^2}$ $\Rightarrow$ $\sqrt[4]{16x^8y^3}$ $\Rightarrow$ Reduce $\Rightarrow$ $2x^2\sqrt[4]{y^3}$

Example: $2\sqrt[3]{27x}$ is read as 2 times $\sqrt[3]{27x}$ When the radical is reduced whatever comes out of the radical is multiplied by what is already out front.

- $2\sqrt[3]{27x}$ $\Rightarrow$ $2\cdot 3\sqrt[3]{x}$ $\Rightarrow$ $6\sqrt[3]{x}$ Done!

Example: Multiply

- $5x\sqrt{4x^3}\cdot 6x^2\sqrt{18x^4}$ $\Leftarrow$ Since this is multiplication we can reshuffle all terms.
- $5x\cdot 6x^2\sqrt{4x^3}\sqrt{18x^4}$ $\Rightarrow$ $30x^3\sqrt{72x^7}$ $\Rightarrow$ Reduce the radical
- $30x^3\sqrt{36\cdot 2x^6x}$ $\Rightarrow$ $30x^3\cdot 6x^3\sqrt{2x}$ $\Rightarrow$ $180x^6\sqrt{2x}$ Done!

Divide

- $\dfrac{\sqrt[3]{162x^7y^{10}}}{\sqrt[3]{6x^4y}}$ $\Leftarrow$ The indexes are the same "3". So, write as one big radical first.
- $\sqrt[3]{\dfrac{162x^7y^{10}}{6x^4y}}$ $\Leftarrow$ Reduce $\Rightarrow$ $\sqrt[3]{27x^3y^9}$ $\Rightarrow$ Simplify $\Rightarrow$ $3xy^3$

Adding/Subtracting radicals: The same as like terms. If all the terms match then combine the number out front.

Example: $8\sqrt{11x}+5\sqrt{11x}-7\sqrt{11x}$ $\Rightarrow$ $(8+5-7)\sqrt{11x}$ $\Rightarrow$ $6\sqrt{11x}$

Some radicals may need to be reduced first before they are added.

Example:

- $5x^2\sqrt{3x}+4\sqrt{12x^5}-2x\sqrt{27x^3}$ $\Leftarrow$ Term do not match
- $5x^2\sqrt{3x}+4\sqrt{4\cdot 3x^4x}-2x\sqrt{9\cdot 3x^2x}$ $\Leftarrow$ Reduce each radical if possible
- $5x^2\sqrt{3x}+4\cdot 2x^2\sqrt{3x}-2\cdot 3x\cdot x\sqrt{3x}$ $\Leftarrow$ Multiply terms out front
- $5x^2\sqrt{3x}+8x^2\sqrt{3x}-6x^2\sqrt{3x}$ $\Leftarrow$ Add like radicals $\Rightarrow$ $7x^2\sqrt{3x}$ Done!

Practice Problems

Perform the indicated operation and reduce all radicals.

1) $\sqrt{128x^3}$

2) $\sqrt{243m^5n^2}$

3) $\sqrt{27a^2}$

4) $\sqrt{2} \cdot \sqrt{8}$

5) $-7\sqrt[3]{3y^2} \cdot \sqrt[3]{18y}$

6) $\sqrt[3]{25p} \cdot \sqrt[3]{125p^3}$

7) $\sqrt{\dfrac{4x^2}{25y^4}}$

8) $\dfrac{\sqrt{48x^3y^7}}{\sqrt{3xy^3}}$

9) $\sqrt[4]{8m^7} \cdot \sqrt[4]{6m^3}$

10) $5\sqrt{7} - 4\sqrt{7} + 2\sqrt{7}$

11) $3x\sqrt{7} + \sqrt{28x^2} - \sqrt{63x^2}$

12) $\sqrt{50} + \sqrt{98} - \sqrt{75} + \sqrt{27}$

Answer Key

1) $8x\sqrt{2x}$

2) $9m^2n\sqrt{3m}$

3) $3a\sqrt{3}$

4) 4

5) $-21y\sqrt[3]{2}$

6) $5p\sqrt[3]{25p}$

7) $\dfrac{2x}{5y^2}$

8) $4xy^2$

9) $2m^2\sqrt[4]{3m^2}$

10) $3\sqrt{7}$

11) $2x\sqrt{7}$

12) $12\sqrt{2} - 2\sqrt{3}$