

Application Problems with Rational Expressions

The applications will involve situations with work rate, variations, water current and speed of wind.

Work rate

Work rate problems usually involve two people that are trying to help each other finish a single job.

Fran can clean the garage in 3 hours, but it takes Angie 4 hours to do the same job. How long would it take them to clean the garage if they worked together?

- ◆ It takes Fran 3 hours to do the work. So, she does $\frac{1}{3}$ of the work each hour.
- ◆ It takes Angie 4 hours to do the work. So, she does $\frac{1}{4}$ of the work each hour.

◆

How long will it take if they help each other is not know so;

- ◆ Let x = The number of hours it takes them to do the work together.
- ◆ Together, they do $\frac{1}{x}$ of the work each hour.

Since, they are working together the sum of the two people individually equals the total.

$$\triangleright \frac{1}{3} + \frac{1}{4} = \frac{1}{x} \quad \text{Solve by clearing fractions with the LCD} = 12x$$

$$\triangleright 12x \cdot \left(\frac{1}{3} + \frac{1}{4} \right) = \frac{1}{x} \cdot 12x \Rightarrow 4x + 3x = 12 \Rightarrow 7x = 12 \Rightarrow x = \frac{12}{7} \approx 1.7$$

Answer: Together it takes about 1.7 hours to finish the job together.

Variation problems

Variation problems should be done in steps. The starting equations will be direct or inverse variations.

- Direct variation $y = kx$ means x and y are directly across from each other.
- Inverse variation $y = \frac{k}{x}$ means that x is inverted on the other side of y .

In both cases k is always in the same position.

Example: If m varies inversely as p , and $m = 30$ when $p = 5$, find m when p is 7.

Step 1 Write the equation, "m varies inversely as p" $\Rightarrow m = \frac{k}{p}$

Step 2 $m = 30$ when $p = 5 \Rightarrow 30 = \frac{k}{5} \Rightarrow k = 150$ ← Use this value of k to solve the final step

Step 3 find m when p is 6 $\Rightarrow m = \frac{150}{6} \Rightarrow m = 25$ Done!

With or Against (wind or current)

Once the equation is set up then the next step will require clearing fractions using the LCD. All solutions must be checked to avoid division by zero.

- Use a table to help organize all the information in the problem. Usually only two columns of the table need to be filled in. The goal is to create the equation using the table.

Example: Garth can row 5 miles per hour in still water. It takes him as long to row 4 miles upstream as 16 miles downstream. How fast is the current?

The equations for rate (r), distance (d), and time (t) are $\Rightarrow d = rt, r = \frac{d}{t}, t = \frac{d}{r}$

- ❖ Let x = speed in still water
- ❖ Let c = speed of the current

The main difference with these problems is rate needs to be expressed using two variables because moving upstream the current is against you and downstream it moves with you.

	Distance	Rate	Time
Upstream		$x - c$	
Downstream		$x + c$	

Fill in the distance column with the numbers from the problem above and the value for speed in still water for x .

	Distance	Rate	Time
Upstream	4	$5 - c$	
Downstream	16	$5 + c$	

Fill in the column for time using the other two columns knowing that $\Rightarrow \text{time} = \frac{\text{distance}}{\text{rate}}$

	Distance	Rate	Time
Upstream	4	$5 - c$	$\frac{4}{5 - c}$
Downstream	16	$5 + c$	$\frac{16}{5 + c}$

"It takes as long ..." from the problem means that the two times are equal to each other. So, the equation can be written as:

$$\circ \quad \frac{4}{5 - c} = \frac{16}{5 + c} \quad \Leftarrow \quad \text{solve by cross-multiplying} \quad \Rightarrow \quad 4(5 + c) = 16(5 - c) \quad \Rightarrow \quad c = 3$$

Answer: The speed of the current is 3 miles per hour.

Note: The speed of the current can not be a negative number, or larger than five. Why?

Practice Problems

Solve each problem and check all solutions. Answer using a complete sentence.

- 1) A boat goes 240 miles downstream in the same time it can go 160 miles upstream. The speed of the current is 5 miles per hour. What is the speed of the boat in still water?

	Distance	Rate	Time
Upstream			
Downstream			

- 2) A plane flies 910 miles with the wind in the same time it can go 660 miles against the wind. The speed of the plane in still air is 305 miles per hour. What is the speed of the wind?

	Distance	Rate	Time
Against			
With			

- 3) A person swims 11 miles downriver in the same time they can swim 7 miles upriver. The speed of the current is 4 miles per hour. Find the speed of the person in still water.

	Distance	Rate	Time
Upstream		$x - c$	
Downstream		$x + c$	

- 4) Kent can paint a certain room in 6 hours, but Kendra needs 4 hours to paint the same room. How long does it take them to paint the room if they work together?
- 5) Marco can build a lap top twice as fast as Cliff. Working together, it takes them 5 hours. How long would it have taken Marco working alone?
- 6) If s varies inversely as t^2 , and $s = 10$ when $t = 2$, find s when t is 10.
- 7) The time (t) traveled by Delmar in a car varies inversely as rate (r). If Delmar drives at a speed of 80 mph in 12 hours, what will be the time to travel if he drives at 60 mph?
- 8) For a given area of a triangle, the base varies inversely as its height. When the height is 10 in the base is 5 in. Find the base if the height is increased to 20 in.

Answer Key

- 1) The speed of the boat in still water is 25 mph. 2) The speed of the wind is about 48.6 mph.
 3) The person can swim 18 mph in still water. 4) I will take 2.4 hours working together.
 5) It takes Marco 7.5 hours. 6) s equals 0.4
 7) It will take 16 hours to drive. 8) The base would be reduced to 2.5 in.