Math 51
Worksheet

How to Solve Equations with Rational Expressions

Solving these equations will require clearing fractions. First find the LCD, distribute to all numerators, and then solve the resulting equation. Sometimes the resulting equation will be a quadratic equation. Check all answers to make sure it does not make the rational expression undefined. Remember division by zero is undefined.

- $\frac{1}{w}+\frac{1}{w-1}=\frac{1}{w^2-w} \Rightarrow \frac{1}{w}+\frac{1}{w-1}=\frac{1}{w(w-1)}$ $\Leftarrow$ factor to find LCD which is $w(w-1)$

- $\frac{1w(w-1)}{w}+\frac{1w(w-1)}{w-1}=\frac{1w(w-1)}{w(w-1)}$ $\Leftarrow$ multiply all the numerators by the LCD

  - Reduce all fractions then solve the resulting equation

- $\frac{1\cancel{w}(w-1)}{\cancel{w}}+\frac{1w\cancel{(w-1)}}{\cancel{(w-1)}}=\frac{1\cancel{w}\cancel{(w-1)}}{\cancel{w}\cancel{(w-1)}} \Rightarrow w-1+w=1 \Rightarrow 2w-1=1 \Rightarrow 2w=2 \Rightarrow w=1$

  - Must check all solutions, some equations may have no solution

- $\frac{1}{(1)}+\frac{1}{(1)-1}=\frac{1}{(1)^2-(1)} \Rightarrow 1+\frac{1}{0}=\frac{1}{0}$ $\Leftarrow$ division by zero so the equation has no solution

When an equation with rational expressions has a single fraction equal to a single fraction clearing fractions can be done by cross multiplying.

- $\frac{2}{x-1}=\frac{3}{x+2}$ $\Leftarrow$ single fraction equal to a single fraction
- $2(x+2)=3(x-1)$ $\Leftarrow$ cross multiply
- $2x+4=3x-3$ $\Leftarrow$ solve the resulting equation
- $x=7$ $\Leftarrow$ must check

Practice Problems

Solve each equation and check all solutions. To find the LCD factor all the denominators first.

1) $\frac{3}{x+3}-\frac{2}{x-3}=\frac{-12}{x^2-9}$

2) $\frac{5}{w-3}=\frac{6}{w+2}$

3) $\frac{4}{b+3}=\frac{1}{b-1}-\frac{1}{b^2+2b-3}$

4) $\frac{4}{k+1}=\frac{3}{k-2}-2$

Mixed review: Perform the indicated operation. If an equation, solve it.

5) $\frac{1}{y}+\frac{1}{y-3}=-\frac{5}{4}$

6) $\frac{\frac{1}{w-1}+\frac{2}{w+2}}{\frac{2}{w+2}-\frac{1}{w-3}}$

7) $\frac{2k^2-3k}{20k^2-5k}\div\frac{2k^2-5k+3}{4k^2+11k-3}$

8) $\frac{1}{x-1}+\frac{1}{2}=\frac{2}{x^2-1}$

9) $\frac{2r}{2r^2+r-3}+\frac{5}{r^2-5r+4}$

10) $\frac{1}{y^2}-\frac{2}{3y}$

Answer Key

1) no solution

2) $w=28$

3) $b=2$

4) $k=3 \quad k=-\frac{5}{2}$

5) $y=\frac{12}{5} \quad y=-1$

6) $\frac{3w^2-9w}{w^2-7w+4}$

7) $\frac{k+3}{5(k-1)}$

8) $x=-3 \quad x\neq 1$

9) $\frac{2r^2+2r+15}{(2r+3)(r-1)(r-4)}$

10) $\frac{3-2y}{3y^2}$