

Combining Rational Expressions

Multiply or Divide rational expressions

- If a division problem flip the expression on the right and change to multiplication.

Example:

$$\circ \frac{3x^2 - 5x - 2}{2x + 14} \div \frac{x^2 + x - 2}{3x + 21} \Rightarrow \frac{3x^2 - 5x - 2}{2x + 14} \cdot \frac{3x + 21}{x^2 + x - 2} \Leftarrow \text{Flip the one on the right.}$$

$$\circ \frac{(3x-1)(x+2)}{2(x+7)} \cdot \frac{3(x+7)}{(x+2)(x-1)} \Leftarrow \text{Factor the numerators and denominators}$$

$$\circ \frac{(3x-1)\cancel{(x+2)}}{2\cancel{(x+7)}} \cdot \frac{3\cancel{(x+7)}}{\cancel{(x+2)}(x-1)} \Leftarrow \text{Divide out common factors}$$

$$\circ \frac{(3x-1)}{2} \cdot \frac{3}{(x-1)} \Rightarrow \frac{3(3x-1)}{2(x-1)} \Leftarrow \text{Multiply across and leave answer in factored form}$$

Add or Subtract rational expressions

Find the LCD (lowest common denominator) by factoring all the denominators first.
List each different denominator factor the greatest number of times it appears in any of the denominators. Rewrite each expression using the LCD.

Example:

$$\frac{4x^2 - 1}{3x^2 - 5x - 2} - \frac{x + 5}{3x - 1} \Rightarrow \frac{4x^2 - 1}{(3x - 1)(x + 2)} - \frac{x + 5}{3x - 1} \Rightarrow \frac{4x^2 - 1}{(3x - 1)(x + 2)} - \frac{(x + 5)(x + 2)}{(3x - 1)(x + 2)}$$

factor *find LCD* *multiply missing factors*

The denominators are the same so Add or Subtract the numerators. Keep denominator in factored form and reduce rational expression to lowest terms if possible.

$$\frac{4x^2 - 1 - (x + 5)(x + 2)}{(3x - 1)(x + 2)} \Rightarrow \frac{4x^2 - 1 - (x^2 + 7x + 10)}{(3x - 1)(x + 2)} \Rightarrow \frac{4x^2 - 1 - x^2 - 7x - 10}{(3x - 1)(x + 2)} \Rightarrow \frac{3x^2 - 7x - 11}{(3x - 1)(x + 2)}$$

Practice Problems

Multiply or Divide rational expressions

$$1) \frac{(x-5)^3}{7x^4 + 14x^3} \cdot \frac{14x^5}{x^2 - 10x + 25}$$

$$2) \frac{5y^6}{3x^2} \div \frac{10y^8}{21x^{11}}$$

$$3) \frac{2x^3 - 6x^2 - 8x + 24}{2x^2 - 8x + 6} \cdot \frac{x^2 - 1}{x^2 - x - 2}$$

$$4) \frac{y-3}{12} \div \frac{3-y}{10}$$

Add or Subtract rational expressions

$$5) \frac{4}{2x^2 + x - 3} + \frac{x}{2x^2 - 5x - 12}$$

$$6) \frac{a+b}{a} - \frac{a+b}{b}$$

$$7) \frac{4}{x+2} - \frac{7}{x-2} + \frac{4x+24}{x^2-4}$$

$$8) \frac{2}{z-4} - \frac{5z}{4-z}$$

Answer Key

$$1) \frac{2x^2(x-5)}{x+2}$$

$$2) \frac{7x^9}{2y^2}$$

$$3) x+2$$

$$4) -\frac{5}{6}$$

$$5) \frac{x^2 + 3x - 16}{(2x+3)(x-1)(x-4)}$$

$$6) \frac{b^2 - a^2}{ab}$$

$$7) \frac{1}{x-2}$$

$$8) \frac{5z+2}{z-4}$$