

Rational Expressions

A rational expression is an expression of the form $\frac{P}{Q}$,
where P and Q are polynomials, with $Q \neq 0$.

Example: $\frac{2x^3 - 7x}{5x^4 - 8x^2}$ Is a rational expression because the top and bottom are polynomials.

Example: $\frac{-2x^{-8} - 7x}{\sqrt{5x^4 - 8x}}$ Is not because of the negative power and the square root.

➤ What makes a rational expression undefined?

A rational expression is undefined when the denominator is equal to zero. To find the values that make a rational expression undefined, set the denominator equal to zero and solve the resulting equation.

Example: $\frac{2x^3 - 7x}{0}$ Is undefined because the zero is in the denominator.

➤ Write the rational expression in lowest terms: $\frac{2x^2 + x - 10}{6x + 15}$

○ Factor the numerator $\Rightarrow \frac{(2x+5)(x-2)}{6x+15}$

○ Factor the denominator $\Rightarrow \frac{(2x+5)(x-2)}{3(2x+5)}$

○ Divide out common factors $\Rightarrow \frac{(2x+5)(x-2)}{3(2x+5)} = \frac{(x-2)}{3} = \frac{x-2}{3}$

Practice Problems

Find any values of the variable for which each rational expression is undefined.

1) $\frac{2x^2 + x - 10}{6x + 24}$

2) $\frac{2m - 3}{m^2 - 9}$

3) $\frac{3r^2 + 7}{2r^2 + r - 10}$

4) $\frac{w^2 + 6w - 10}{5w^2 + 10}$

Write each rational expression in lowest terms.

5) $\frac{21x^7}{3x^2}$

6) $\frac{6x + 12}{4x + 8}$

7) $\frac{2x^3 + 6x^2 - 7x - 21}{x^2 + 5x + 6}$ use grouping

8) $\frac{x^2 + 3x - 18}{x^2 + 8x + 12}$

9) $\frac{p^3 - 27}{p - 3}$ use difference of cubes

10) $\frac{2 - 5x}{5x - 2}$

Answer Key

1) $x \neq -4$

2) $m \neq 3 \quad m \neq -3$

3) $r \neq -\frac{5}{2} \quad r \neq 2$

4) It is never undefined

5) $7x^5$

6) $\frac{3}{2}$

7) $\frac{2x^2 - 7}{x + 2}$

8) $\frac{x - 3}{x + 2}$

9) $p^2 + 3p + 9$

10) -1