Logarithms and Exponentials

A *logarithmic* function is the inverse of an *exponential* function, and an *exponential* function is the inverse of a *logarithmic* function. For example,

$f(x) = 2^x$ inverse ⟶ $f^{-1}(x) = \log_2 x$
(exponential) *(logarithm)*

$f(x) = \log_2 x$ inverse ⟶ $f^{-1}(x) = 2^x$
(logarithm) *(exponential)*

Note: An exponential function is one with a variable in the exponent. So, $f(x) = b^x$ (where b is a positive constant, $b \neq 1$) is the exponential function, base b. The function $f(x) = x^2$ is NOT an exponential function.

How to *read* logarithms ("logs"):

When you have ... $\mathbf{\log_5 x}$ ... it is read, *"the log, base 5, of x."*
Or when you have ... $\mathbf{\log_2 x^4}$ ... it is read, *"the log, base 2, of x to the power of 4."*

*Note: In the expression $\log_b c$, **b** is the "base" and **c** is called the "argument." ($b > 0$, $c > 0$)*

Converting LOGARITHMS into EXPONENTIALS and EXPONENTIALS into LOGARITHMS:

*"**a** is equal to the **log,** base **b**, of **c**"* *"**b** to the power of **a** is equal to **c**"*

$a = \log_b c$ ⟵ *converts to* ⟶ $b^a = c$
(logarithmic) *(exponential)*

*Note: In the logarithmic equation, the base is **b**. This corresponds to the base **b** in the exponential equation. In the exponential equation, the exponent is **a**, and this corresponds to what the entire log is equal to in the logarithmic equation.* The logarithm is an exponent. *If we can remember how these "positions" of **a**, **b**, **c** convert from logarithms to exponentials and back again, then rewriting logs as exponentials or exponentials as logs won't be a problem.*

ex: $9 = 3^x$ *converts to* ⟶ $x = \log_3 9$
$y = \log_3 27$ *converts to* ⟶ $3^y = 27$

LOGARITHMIC and EXPONENTIAL Equality:

If ... $\mathbf{b^x = b^y}$ ⟶ The bases "**b**" are the same ($b \neq -1, 0, 1$).
then ... $\mathbf{x = y}$ ⟶ We may equate exponents.
If ... $\mathbf{\log_a x = \log_a y}$ ⟶ The bases "**a**" are the same ($x, y, a > 0$).
then ... $\mathbf{x = y}$ ⟶ We may equate the arguments.

ex: $4^{(x+1)} = 4^{(2x)}$	Since the bases are both "4," we may equate exponents.
$x + 1 = 2x$	Subtract "x" from both sides.
$\mathbf{1 = x}$	Answer.

ex: $\log_2 (x + 2) = \log_2 4$	Same base means we may equate arguments.
$x + 2 = 4$	Subtract "2" from both sides.
$\mathbf{x = 2}$	Answer. The argument must be greater than zero, so $(x + 2) > 0$ … $(2 + 2) > 0$ … $4 > 0$ … answer checks.

 revised 4/10/09

Logarithmic Properties: The following is a list of some logarithmic properties that may be useful.

PRODUCT RULE for LOGS

$\mathbf{\log_b(MN) = (\log_b M) + (\log_b N)}$ → For any positive numbers M, N, and b (b ≠ 1).

ex: $\log_2 (xy) = (\log_2 x) + (\log_2 y)$

QUOTIENT RULE for LOGS

$\mathbf{\log_b \frac{M}{N} = (\log_b M) - (\log_b N)}$ → For any positive numbers M, N, and b (b ≠ 1).

ex: $\log_{10} \frac{x^2}{y} = (\log_{10} x^2) - (\log_{10} y)$

POWER RULE for LOGS

$\mathbf{\log_b M^p = p (\log_b M)}$ → For any positive number M and b (b ≠ 1), and any real number p.

ex: $\log_5 25^x = x (\log_5 25)$

LOG of a BASE raised to a POWER

$\mathbf{\log_b b^k = k}$ → For any base b.

ex: $\log_2 2^x = x$

CHANGE of BASE

$\mathbf{\log_b M = \frac{\log_a M}{\log_a b}}$ → For any logarithmic bases a and b, and any positive number M.

ex: $\log_3 5 = \frac{\ln 5}{\ln 3}$

Note also: You CANNOT take the log of a negative number.
$\log_{10} (-4)$ = undefined

One more thing: Remember "log" is not distributive ...

$\log (M \pm N) \neq \log M \pm \log N$
$\log (MN) \neq (\log M)(\log N)$
$\log \left(\frac{M}{N}\right) \neq \left(\frac{\log M}{\log N}\right)$

*Note: When using the change of base formula, we will often change to base 10 (so in the change of base formula, **a** = 10). In this way, you can use the* log *button on your calculator to solve the problem. A log with base 10 is called the "common log" and is written without the ten, so $\log_{10} 5$ is written log 5. The "natural log" is another log that comes up often. The natural log is just a log with base "e" ("e" is a constant equal to approximately 2.718281828). Natural logs use ln__ instead of $\log_e$__, so $\log_e 4$ is written ln 4. The* ln *button is also on most calculators, so you could change to base "e" if you choose.*

ex: log x = 3.8	Rewrite the common log as a power of ten.	ex: ln x = 2	Rewrite the natural log as a power of e.
$\mathbf{10^{3.8} = x}$	Exact answer.	$\mathbf{e^2 = x}$	Exact answer.
6309.57 ≈ x	Approximate answer. Checks since 6309.57 > 0	**7.39 ≈ x**	Approximate answer. Checks since 7.39 > 0

revised 4/10/09