Graphing Lines / Finding Equations of Lines

There are two ways to graph a line.

1.) We can plot points (ordered pairs); we will need at least two ordered pairs to graph the line.

2.) We can put the equation of the line into slope-intercept form y = mx + b, plot the y-intercept (0, b) and starting from the y-intercept, count out the slope (m = rise / run) to get the second point.

We will return to graphing later. First, let's discuss some formulas and the forms for equations of lines.

$$\textbf{slope}\,(m) = \frac{rise}{run} = \frac{\text{change in } y}{\text{change in } x} = \frac{y_2 - y_1}{x_2 - x_1}$$ ⟶ Parallel lines have the same slope ($m_1 = \frac{1}{2}$, $m_2 = \frac{1}{2}$). Perpendicular lines have negative reciprocal slopes ($m_1 = 2/3$, $m_2 = -3/2$). Perpendicular lines are lines that form 90-degree angles where they cross.

Equations of a line:

Slope-intercept form: y = mx + b ⟶ m = slope, b = y coordinate of y-intercept

Point-slope form: $(y - y_1) = m\,(x - x_1)$ ⟶ (x_1, y_1) is a point on the line

Standard form: ax + by = c ⟶ a, b, c are integers

Vertical line: x = constant ⟶ m = undefined

Horizontal line: y = constant ⟶ m = 0

Slope formula: Used to find the slope when two points, two ordered pairs (x, y), are given.

　　ex: Find the slope of the line that goes through the points (1, 0) and (4, 2).

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{2 - 0}{4 - 1} = \frac{2}{3}$$ ⟶ Plug the two ordered pairs into the formula and simplify.

<u>Point-slope form</u>: Used when the slope (m) is known and one point (x_1, y_1) is known and you are asked to find the equation of a line. Also used when given two points and asked for the equation of a line (in the latter case, you will use the slope formula first).

   ex: The slope of a line is 2 and the line goes through the point (4, 3). Find the equation of the line.

   $(y - y_1) = m (x - x_1)$
   $(y - 3) = 2 (x - 4)$ ————▶ Plug in the given values for slope and the ordered pair.
   $y - 3 = 2x - 8$ ————▶ Distribute the 2.
   $y = 2x - 5$ ————▶ We added a three to each side to put the equation in **slope-intercept form**. The problem didn't specify what form to put our answer in, but this is a common form.

   $2x - y = 5$ ————▶ With some rearranging we may put the line in **standard form**, if necessary.

<u>Procedure for finding x and y-intercepts of a line:</u> Two important points on a line are the points where the line crosses the x and the y-axes. These x and y-intercepts are often labeled on a graph.

To find the x-intercept: set $y = 0$ and solve for x
To find the y-intercept: set $x = 0$ and solve for y

   ex: Given the line $x - 2y = -4$, find the x and y-intercepts.

   $x - 2 (0) = -4$ ————▶ Set the $y = 0$
   $x - 0 = -4$
   $x = -4$ ————▶ The x coordinate of the x-intercept is -4.

   $0 - 2y = -4$ ————▶ Set the $x = 0$
   $-2y = -4$
   $y = 2$ ————▶ By dividing both sides by -2, we see that the y coordinate of the y-intercept is 2.
   x-intercept: $(-4, 0)$ ————▶ Answer
   y-intercept: $(0, 2)$

Note: If the line is in slope intercept form, the y-intercept is easy to identify since in the form, y = mx + b, the y-intercept is (0, b). But if the line is in some other form, the above procedure may be used.

Graphing lines, as mentioned above, is usually done in one of two ways. Let's take two lines and graph each one with different methods.

Plotting points: This technique will involve the creation of a table of ordered pairs (x, y). We will then plot these points on the Cartesian coordinate plane (the Cartesian coordinate plane is the formal name for the xy-plane that we're used to).

ex: Graph the line, $3x - 2y = 6$.

x	y
0	
	0
4	

We have constructed a table to identify points on the line. We will arbitrarily pick x and y values and find their corresponding coordinates. We have decided to find the x and y-intercepts (notice that we have set $x = 0$ in one point and $y = 0$ in the other). We only need two points to graph a line, but will find one more for demonstration.

$3(\mathbf{0}) - 2y = 6 \longrightarrow$ Set $x = 0$
$-2y = 6$
$\mathbf{y = -3} \longrightarrow$ We find one point on the line is **(0, − 3)**.

$3x - 2(\mathbf{0}) = 6 \longrightarrow$ Set $y = 0$
$3x = 6$
$\mathbf{x = 2} \longrightarrow$ We find a second point on the line **(2, 0)**.
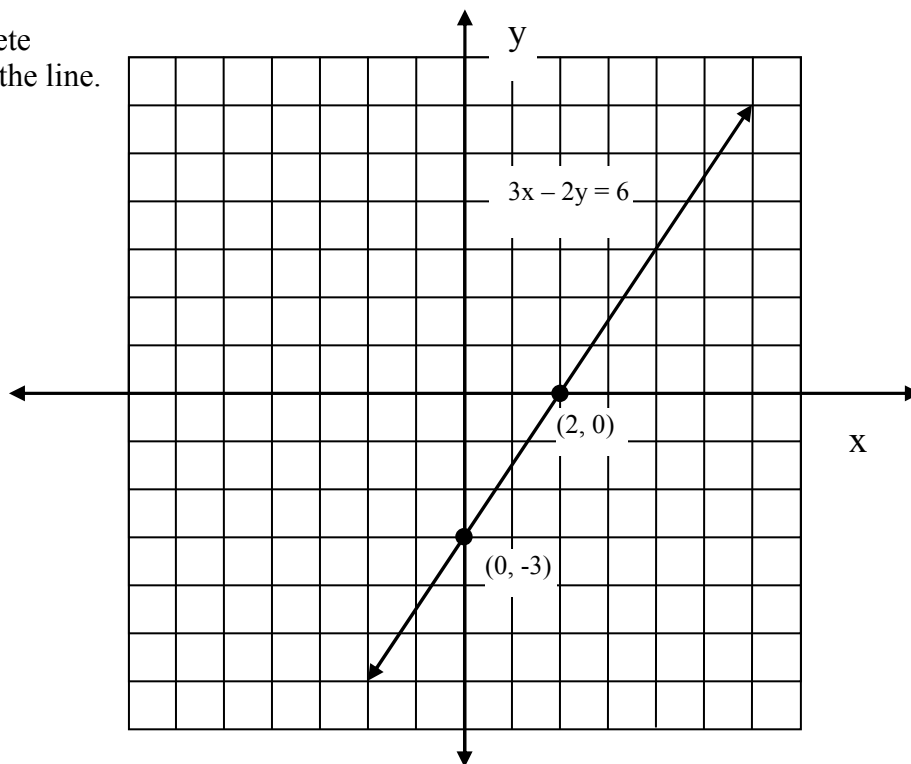
$3(\mathbf{4}) - 2y = 6 \longrightarrow$ Set $x = 4$
$12 - 2y = 6$
$-2y = -6$
$\mathbf{y = 3} \longrightarrow$ A third point on the line is **(4, 3)**

Now we can complete the table and graph the line.

x	y
0	− 3
2	0
4	3

<u>Slope-intercept form:</u> To use this method to graph a line, we will first put the line into the slope-intercept form (y = mx + b). Then we will plot the y-intercept (0, b) and starting from this point we will "count" over to the next point using the slope. Remember that the slope is the rise divided by the run. The *rise* is the change in the y-coordinate and the *run* is the change in the x-coordinate.

ex: Graph the line $2x - 4y = 8$

$-4y = -2x + 8$ ⟶ We begin to put the equation in slope-intercept form by first subtracting the "2x" term from both sides. We need to isolate the y, but first we will isolate the term that contains the y.

$y = \dfrac{1}{2}x - 2$ ⟶ To get the y by itself and put into slope intercept form, we divided both sides by – 4. Now that the equation is in this form, it is easy to identify the y-intercept and the slope of this line. **The y-intercept is (0, – 2) and the slope is ½.**

Now we are ready to graph the line.

First plot the y-intercept (0, – 2).

Then *rise* one (since the one is positive, we will go **up** one unit) and *run* two (since the two is positive, we will go to the **right** two units).

This takes us to another point on the line (2, – 1).

Note: $\dfrac{-1}{-2} = \dfrac{1}{2}$ *This suggests that if we were to "count" out a rise of –1 and a run of –2, we should still get a point on the line (to* rise *–1, we will go **down** one unit and to* run *–2 we will go to the **left** 2 units). Note that this point (–2, –3) is located on the same line.*

Note also: If the slope had been a negative number like –3/4, we could go down 3 and to the right 4, or we could go up 3 and to the left 4. This is true because $\dfrac{-3}{4} = \dfrac{3}{-4}$. *If we begin at a point on the line (probably the y-intercept), either of these counting methods will give another point on the line.*

(graph showing line $2x - 4y = 8$ with points $(0, -2)$ and $(2, -1)$ labeled; "(2, –1) Going up one and to the right two brings us to this point.")