Properties of Exponents

An exponent (also called power or degree) tells us how many times the base will be multiplied by itself. For example x^5 , the exponent is 5 and the base is x. This means that the variable x will be multiplied by itself 5 times. You can also think of this as x to the fifth power.

Below is a list of properties of exponents:

Properties	General Form	Application	Example
Product Rule Same base add exponents	a ^m a ⁿ	a ^{m+n}	$x^5 x^3 = x^{5+3} = x^8$
Quotient Rule Same base subtract exponents	$\frac{a^m}{a^n}$	a^{m-n}	$\frac{x^9}{x^5} = x^{9-5} = x^4$
Power Rule I Power raised to a power multiply exponents.	$(a^m)^n$	a ^{mn}	$(x^3)^4 = x^{3 \cdot 4} = x^{12}$
Power Rule II Product to power distribute to each base	(<i>ab</i>) ^{<i>m</i>}	$a^m a^n$	$(4x^3)^2 = 4^2 x^{3 \cdot 2} = 16x^6$
Negative Exponent I Flip and change sign to positive	a ^{-m}	$\frac{1}{a^m}$	$x^{-3} = \frac{1}{x^3}$
Negative Exponent II Flip and change sign to positive	$\frac{1}{a^{-m}}$	a^m	$\frac{1}{x^{-5}} = x^5$
Zero Exponent Anything to the zero power (except 0) is one	a ⁰	$a^{0} = 1$	$(-4x)^0 = 1$

• It is important to note that none of these applications can occur if the bases are not the same.

For example, $\frac{x^5}{y^3}$ cannot be simplified.

Using a Combination of Rules

At one point, you may be asked to use a combination of these properties.

Example:

•
$$\frac{(2^{3}y^{2})^{5}}{2^{10}y^{16}} \rightarrow \text{Power Rule}$$
•
$$\frac{2^{3 \cdot 5}y^{2 \cdot 5}}{2^{10}y^{16}}$$
•
$$\frac{2^{15}y^{10}}{2^{10}y^{16}} \rightarrow \text{Quotient Rule}$$
•
$$2^{15-10}y^{10-16}$$
•
$$2^{5}y^{-6} \rightarrow \text{Negative Exponent}$$
•
$$\frac{32}{y^{6}}$$

Example:

•
$$\left(\frac{p^{-4}q}{r^{-3}}\right)^{-3} \rightarrow \text{Power Rule}$$

•
$$\frac{p^{-4 \cdot -3} q^{1 \cdot -3}}{r^{-3 \cdot -3}}$$
 Note: When a base does not have an exponent there is really a one as the power. So that, q is understood as q^1

•
$$\frac{p^{12}q^{-3}}{r^9} \rightarrow \text{Negative Exponents}$$

$$\bullet \quad \frac{p^{12}}{q^3 r^9}$$