Math 181 **Chris McDonald**

SERIES

Convergence

1. Geometric:

$$\sum_{n=1}^{\infty} ar^{n-1} \text{ or } \sum_{n=0}^{\infty} ar^{n}, \text{ converges to } \frac{a}{1-r} \text{ if } |r|<1.$$

2. Harmonic:

$$\sum_{n=1}^{\infty} \frac{1}{n} \text{ diverges;}$$

Useful when using the Comparison or Limit Comparison Tests.

3. P-Series:

$$\sum_{n=1}^{\infty} \frac{1}{n^{p}} \text{ converges if } p>1, \text{ diverges if } p \leq 1.$$

4. Test for Divergence:

If $\lim_{n\to\infty} a_n \neq 0$ or $\lim_{n\to\infty} a_n$ does not exist, then the series $\sum_{n=1}^{\infty} a_n$ diverges.

5. The Comparison Test:

If $\sum_{n=1}^{\infty} a_n$ and $\sum_{n=1}^{\infty} b_n$ are series with the positive terms and $a_n \leq b_n$ then,

if $\sum_{n=1}^{\infty} a_n$ diverges then $\sum_{n=1}^{\infty} b_n$ diverges,

if $\sum_{n=1}^{\infty} b_n$ converges then $\sum_{n=1}^{\infty} a_n$ converges.

Useful if the series has a form similar to the geometric or p-series.

6. The Limit Comparison Test:

If $\sum_{n=1}^{\infty} a_n$ and $\sum_{n=1}^{\infty} b_n$ are series with the positive terms and $\lim_{n\to\infty} \frac{a_n}{b_n} = c$, where c is a finite number and $c>0$, then either both series converge or both diverge.

Useful if the series has a form similar to the geometric or p-series.

7. Alternating Series Test:

If $\sum_{n=1}^{\infty}(-1)^{n-1} b_n$ where $b_{n+1} \le b_n$ for all n and $\lim_{n\to\infty} b_n = 0$, then the series is convergent.

8. $\sum_{n=1}^{\infty} a_n$ is **Absolutely Convergent** provided $\sum_{n=1}^{\infty} |a_n|$ is convergent.

9. $\sum_{n=1}^{\infty} a_n$ is **Conditionally Convergent** if it converges, but not absolutely.

10. The Ratio Test:

If $\lim_{n\to\infty}\left|\dfrac{a_{n+1}}{a_n}\right| = L$, then the series $\sum_{n=1}^{\infty} a_n$ converges absolutely if $L < 1$, diverges if $L > 1$, if $L = 1$ the test is inconclusive.

Useful if a_n has factorial terms or nth powers of constants

11. The Root Test:

If $\lim_{n\to\infty}\sqrt[n]{|a_n|} = L$, then the series $\sum_{n=1}^{\infty} a_n$ converges absolutely if $L < 1$, diverges if $L > 1$, if $L = 1$ the test is inconclusive.

Useful for terms of the form $(a_n)^n$

12. Integral Test:

If f is continuous, positive, decreasing and $a_n = f(n)$ then,

if $\int_1^{\infty} f(x)\,dx$ converges, then $\sum_{n=1}^{\infty} a_n$ converges,

if $\int_1^{\infty} f(x)\,dx$ diverges, then $\sum_{n=1}^{\infty} a_n$ diverges.

Useful if $\int_1^{\infty} f(x)\,dx$ is easily evaluated.