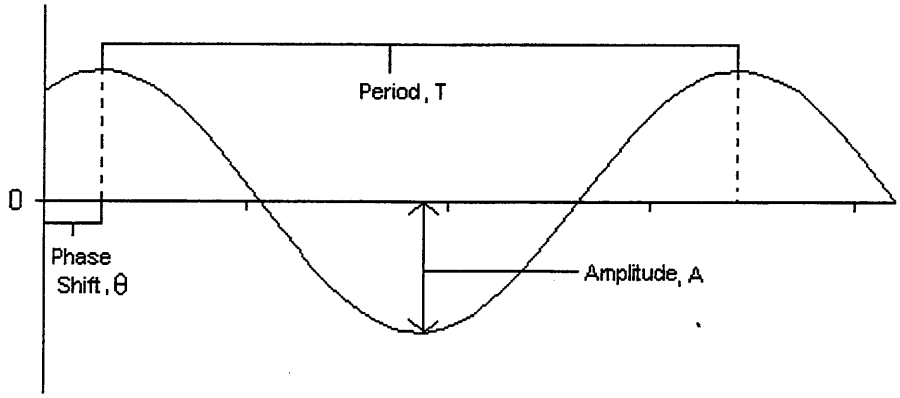
Graphing sine and cosine functions

All sine and cosine graphs can be characterized by four values: the amplitude, the period, the phase shift, and the vertical translation. All four of these values can be obtained from the graph or from the equation of the function. Sine and cosine functions are written as:

$$f(x) = A \sin (Bx + C) + D \qquad \text{or} \qquad g(x) = A \cos (Bx + C) + D$$



From earlier chapters, we know that $\sin(x) = \cos(x-90°)$, so all of these formulas and ideas will hold true for both functions.

The amplitude is equal to one-half the vertical distance from the minimum value to the maximum value of the graph and is equal to A in the function's equation.

The period of a graph is the distance from *any* point on the graph to the corresponding point on the graph in the next cycle. Maximums and minimums are easy choices, but any point can be used to find the period from the graph. From the equation of a function, the period can be found by the following formula:

$$\text{The period, } T = \frac{2\pi}{B} \text{ in radians, or } T = \frac{360°}{B} \text{ in degrees.}$$

The vertical translation of a graph is the amount by which it is shifted up or down. The standard sine and cosine graphs both have an average value of 0, but the graphs can move away from this position. The amount by which this value is shifted is equal to D in the function's equation. From a graph, it is found by taking the *average* of the minimum and maximum values.

The phase shift, θ, is often the most troublesome to find. It is the amount by which the whole graph is shifted left or right from its standard position. The phase shift is found by using the following formula:

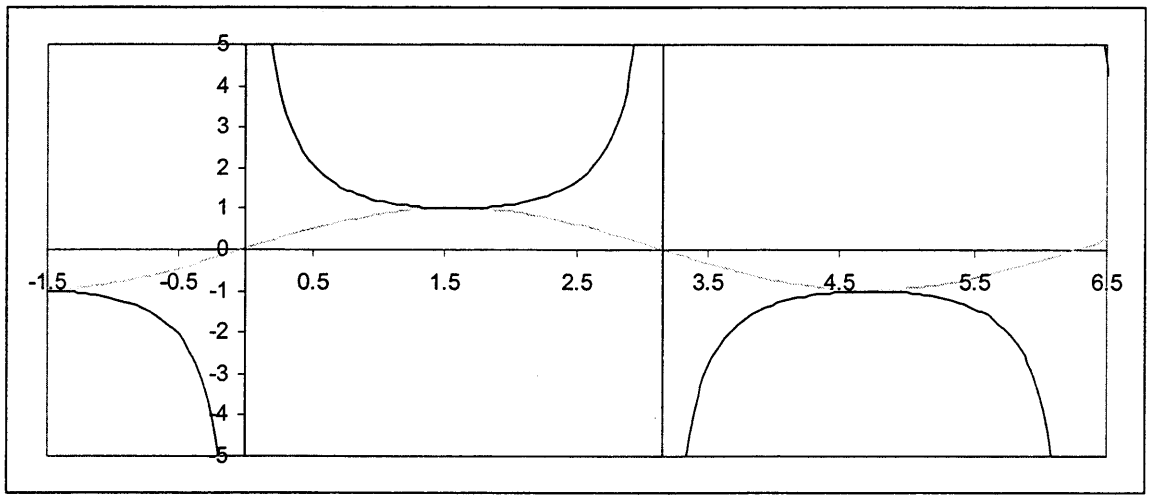
$$\theta = -\frac{C}{B}$$

*Note: It is best, though not always required, for the phase shift and Bx to have the same units.(i.e., it is awkward to have Bx expressed in radians and θ in degrees.)

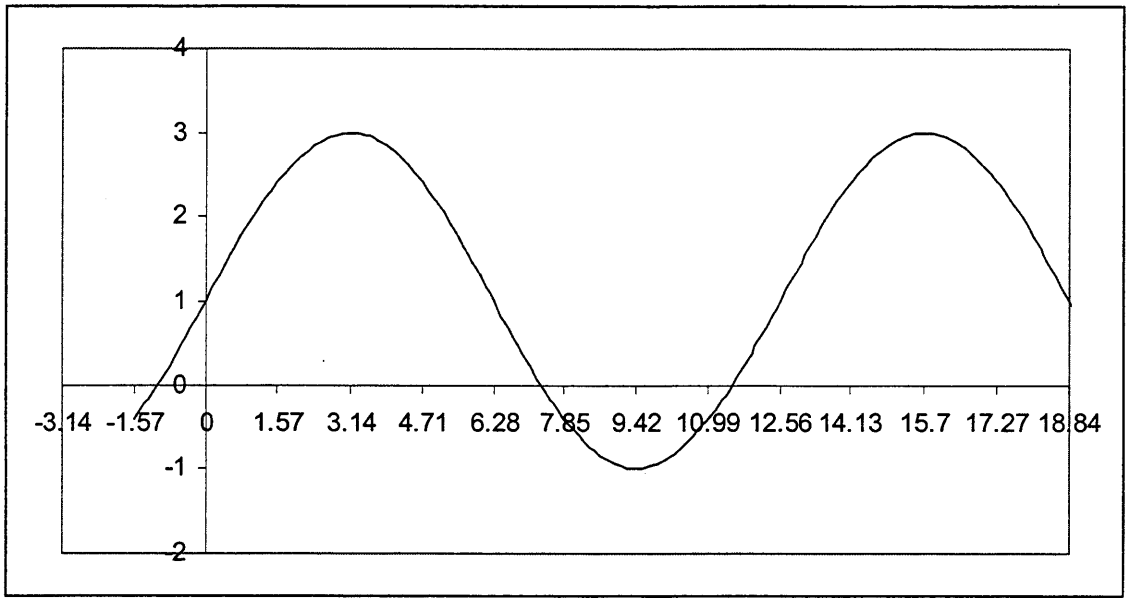
Graphing sine and cosine functions

Graphing Reciprocal Trigonometric Functions

The cosecant and secant graphs are not very easy to draw, but they become much easier if we first draw the graphs of their related sine or cosine graphs. To turn a sine into a cosecant or a cosine into a secant, simply take the reciprocals of a few points. The zeroes of the sine and cosine graph become asymptotes of the reciprocal graphs, and the maximums become minimums and minimums become maximums.



Ex. Given the following sine graph, find an equation.



One-half the vertical distance from the min to the max is ½*(3-¯1) = 2, so A = 2,
The period is (12.7-3.14) = 4π, so B = 2π/4π = ½,
The vertical translation is (3+ -1) / 2 = 1, so D = 1 and
The Phase shift is 0, so C = 0. (Note that the vertical shift sometimes causes it to look like we have a phase shift, but this is not the case)
Therefore, our equation is $2 \sin (\frac{1}{2} x) + 1$.