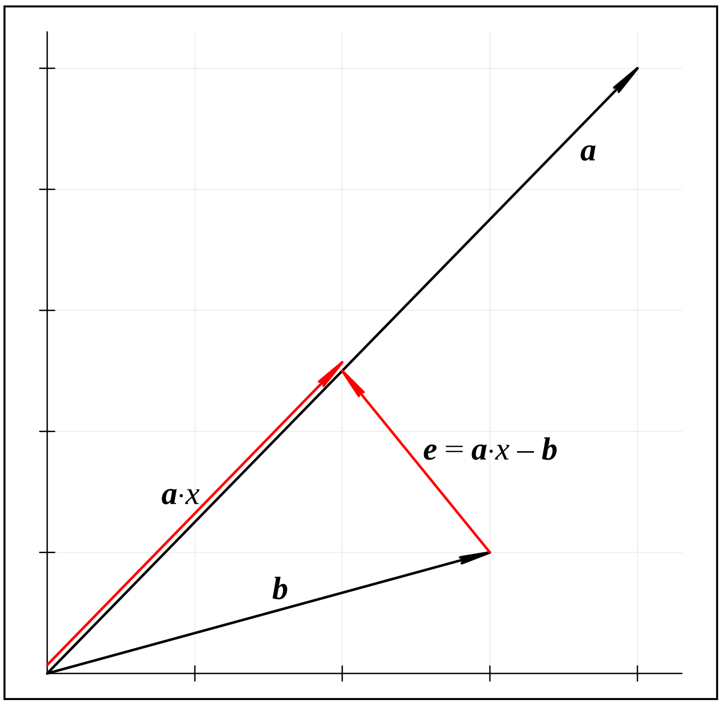
Practical Intermediate Algebra

Detailed Textbook Outline



Mathematics Department
Mt. San Antonio College

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Outline for Practical Intermediate Algebra Text

Chapter One – Scientific Essentials

Section 1.1: Ratios and Proportions

1. Intro and section overview
2. Ratios, percents, relative frequencies (dimensionless), rates (with dimensions)
    a. Examples
        i. A can of a particular brand of motor costs $4.50 for 32 ounces. What is the cost per ounce?
        ii. In the Fahrenheit scale there are 212° – 32° = 180° degrees difference between freezing and boiling temperatures for water. In the Celcius scale, there are 100° – 0° = 100° degrees difference between these two temperature states. How many degrees in Fahrenheit are there for every degree Celsius?
        iii. A 195 pound patient is to receive 254 mL of a given intravenous medication per hour. How many milliliters per pound is the patient receiving per hour?
        iv. A car travels 527 miles in 8.5 hours. What is the average speed in miles per hour?
3. Proportions
    a. Defined
        i. Defined in terms of rates
        ii. Defined in terms of means and extremes
    b. Examples
        i. Dosages depending on weight
            1. A certain children's ibuprophen medication indicates that 3 teaspoons are to be given to a child weighing 95 pounds. If this dosage ratio is to be maintained for all children, what dose should be given to a child weighing 40 pounds (to the nearest $1/10^{th}$ teaspoon).
            2. An animal antibiotic medication dosage is 250 mg per 50 pounds. What dosage should be given to an animal weighing 326 pounds?
            3. A children's acetaminophen pain reliever is to be given in dosages of 2 tablets per 24 pounds. How many tablets should be given to a child weighing 65 pounds? Round your answer to the nearest ¼ tablet.
        ii. Similar triangles
            1. Suppose that two triangles are similar, and that the first has sides 6, 12, and 9. The second triangle has sides x, y, and z, respectively. If $x = 10$, find y and z.
        iii. Proportions involving given rates
            1. Cost
            2. Speed
            3. Work rate
        iv. Proportions with Percentages
            1. 35% of students in our class are male. If there are 14 males in the class, how many females are there?
            2. 45% of the trees on our property are pine trees. The rest of the trees are oaks, and there are 22 of those. How many oaks are there?
            3. A window cleaning solution calls for 10% ammonia and 90% water. If we have 6 cups of ammonia, how many liters of window cleaner can we make?

Section 1.2: Unit Conversion

1. Intro and section overview
2. Conversion factors and units
   - *a.* Listing of more important conversions
     - *i.* Length: inches, feet, yards, miles, meters, AU (astronomical unit), …
     - *ii.* Area: acres, square miles, square meters, …
     - *iii.* Volume: gallons, liters, …
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     - *vi.* Time
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   - *c.* Proportion problems with unit conversions
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     - *ii.* If a certain medication is to be given at 3 tsp per 80 lbs, how many mL should a 205 lb patient take?
     - *iii.* If a car drives 65 miles per hour, in how many minutes will the car drive 20 miles?
     - *iv.* If a given medication for animals is to be given in doses of 30 mL per 100 pounds, how many ounces should be given to an animal weighing 50 kg?
4. Special case: converting degrees Celsius to/from degrees Fahrenheit

Section 1.3: Numeracy

1. Intro and section overview
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3. Scientific Notation and Orders of Magnitude
    - *a.* Example. Convert to scientific notation: 352.916
    - *b.* Example. Convert to scientific notation: –0.003164
    - *c.* Example. Convert from scientific notation: -2.0801×10^{6}
    - *d.* Example. Convert from scientific notation: 4.9851×10^{-7}
4. Estimates and Reasonable Results
    - *a.* Reasonable dosages.
        - *i.* If an adult dose of a particular medication is 250mg, would it be reasonable to give an infant, aged one year or less, a 125mg dose? Do no computations, but explain your answer.
        - *ii.* If 210 pound person's dose of a particular medication is 40 mL, is it reasonable to assume that a 150 pound person's dose would be around 3 tsp? Do no computations, but explain your answer.
    - *b.* Reasonable amounts
        - *i.* If a car can drive from Los Angeles to San Luis Obispo on 12 gallons of gas, is it reasonable to assume that a fully loaded semi-truck could do the same on 90 liters of gas? Do no computations, but explain your answer.
        - *ii.* For a $356 restaurant bill, would a $28 tip be unusually large or small? Do no computations, but explain your answer.
        - *iii.* A person with no children who makes $50,000 per year pays $1000 in Federal income tax. Is this a reasonable amount, too high or too low? Explain your answer.
        - *iv.* If a car gets 25 miles per gallon of gas on the highway, is it reasonable to assume that the car can drive 12 miles from San Bernardino to Lake Arrowhead (a 4000 foot gain in elevation) on ½ gallon of gas? If no, would the car use more or less than ½ gallon?
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      ii. $-2^2 =$
      iii. $1 + 2/(3+4) =$
      iv. $\dfrac{(15-12)^2}{12} =$
      v. $\dfrac{25-23.4}{3.2/\sqrt{26}} =$
      vi. $\dfrac{12.7-14.2}{\sqrt{\dfrac{3.6^2}{24}+\dfrac{4.5^2}{32}}} =$
      vii. $\dfrac{4\cdot 2.1^2+12\cdot 4.3^2}{4+12-2} =$
   b. Combining like terms and the distributive property: $ax+bx=(a+b)x$
      i. $2x+9x=$
      ii. $-24y-7y=$
      iii. $2\sqrt{x}+7\sqrt{x}=$
      iv. $3\sqrt{5}-7\sqrt{5}=$
      v. $-3e^{2x}+11e^{2x}=$
      vi. $14y\sqrt[3]{5x}+5y\sqrt[3]{5x}=$
3. Properties of exponents
   a. Examples of simplifying products involving exponential expressions
      i. $x^2 y\cdot x^{-4}y^2$
      ii. $a^{-3}bc^5\cdot a^6b^{-4}c^2$
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      iv. $9.32\times 10^{-3}\cdot 4.67\times 10^6$
4. Properties of radicals
   a. Simplifying square roots containing perfect squares
      i. $2\sqrt{72}$
      ii. $4\sqrt{300}$
   b. Combining like radicals after simplifying
      i. $4\sqrt{20}+3\sqrt{45}$
   c. Examples of simplifying products and ratios involving radical and exponential expressions
      i. $3\sqrt{18}\sqrt{8}=$
      ii. $-2\sqrt{10}\sqrt{20}=$
      iii. $x^2\sqrt{x^3}\sqrt{x^5}=$
      iv. $\sqrt{\dfrac{a^{12}}{b^5}}=$

v. $\sqrt{\dfrac{x^2 y^4}{z^{-6} w^8}} =$

5. Reducing/simplifying rational expressions
   a. Examples of reducing rational expressions – no factoring required

i. $\dfrac{25x}{15x} =$

ii. $\dfrac{12x}{8x^2 y} =$

iii. $\dfrac{18x^3}{27x^5} =$

iv. $\dfrac{14z^7}{21z^3}$

v. $\sqrt{\dfrac{x^7}{x^5}} =$

vi. $\sqrt{\dfrac{x^3 y^{-5}}{xy^7}} =$

vii. $\dfrac{3.158\times10^{-3}}{2.716\times10^{-5}} =$

viii. $\dfrac{253.19\times10^{12}}{1281.85\times10^{20}} =$

ix. $\sqrt{\dfrac{2.74\times10^{17}}{3.19\times10^{5}}} =$

Section 1.5: Isolating Variables in Literal Equations

1. Intro and section overview
2. Properties of equality & field axioms
3. Isolating variables by addition, subtraction, multiplication, and division rules.
    - *a.* Perimeter: The perimeter of a rectangle is $P = 2l + 2w$. Solve for *w*.
    - *b.* Degrees *C* to/from degrees *F*: To convert from degrees Celsius to degrees Fahrenheit the formula is $F = \frac{9}{5}C + 32$. Solve for *C*.
    - *c.* Triangle area: The area of a triangle is given by $A = \frac{1}{2}bh$. Solve for *b*.
    - *d.* Ideal gas law: The ideal gas law from chemistry is $PV = nRT$. Solve for *T*.
    - *e.* Distance/rate/time: Distance under constant rate of speed is given by $D = RT$. Solve for *R*.
    - *f.* Simple Interest: Simple interest is computed using the formula: $I = P \cdot r \cdot t$. Solve for *r*.
4. Solving literal equations by extracting roots with the square root property
    - *a.* Area examples – squares. The area of a square is $A = x^2$. Solve for *x*.
    - *b.* Falling objects: The distance traveled by a falling object in *t* seconds is $y = \frac{1}{2} \cdot g \cdot t^2$. Solve for *t*.
    - *c.* Area example: The area of a circle of radius r is given by $A = \pi r^2$. Solve for *r*.
    - *d.* Sound/light intensity
    - *e.* Pythagorean theorem: The Pythagorean theorem is $a^2 + b^2 = c^2$, where *a* and *b* are the leg lengths of a right triangle, and *c* is the length of the hypotenuse. Solve for *b*.

Section 1.6: Functions and Graphs

1. Intro and section overview
2. Definition of function: notation and evaluation by substitution
    a. Examples – substituting numbers
        i. $f(x)=2x+3$
        ii. $g(x)=\sqrt{x+1}$
        iii. $h(x)=\frac{4}{x^2}$
        iv. $f(x)=x^2$
3. Domain and range
    a. Process for determining domain
    b. Examples
        i. $f(x)=2x+3$
        ii. $g(x)=\sqrt{x+1}$
        iii. $h(x)=\frac{4}{x^2}$
        iv. $f(x)=x^2$
4. Graphing & intercepts
    a. Point-to-point process for graphing
    b. Process for finding intercepts
    c. Examples
        i. $f(x)=2x+3$
        ii. $g(x)=\sqrt{x+1}$
        iii. $h(x)=\frac{4}{x^2}$
        iv. $f(x)=x^2$
5. Scatter plots
    a. Example: The FBI and Bureau of Alcohol, Tobacco and Firearms have compiled data with the goal of determining whether there is a relationship between the number of registered automatic weapons in a given city with that city's murder rate. These data include the following for 8 different cities.

Automatic Weapons (in Thousands)	*Murder Rate* (Murders per 100,000 people)
11.6	13.1
8.3	10.6
3.6	10.1
0.6	4.4
6.9	11.5
2.5	6.6
2.4	3.6
2.6	5.3

6. Histograms & relative frequency histograms
    a. Example: The journal *Environmental Concentration and Toxicology* published the article "Trace Metals in Sea Scallops" (vol. 19, pp. 326 - 1334), and gave the following cadmium amounts (in mg) in sea scallops observed at a number of different stations in North Atlantic waters. The values are: 5.1, 14.4, 14.7, 10.8, 6.5, 5.7, 7.7, 14.1, 9.5, 3.7, 8.9, 7.9, 7.9, 4.5, 10.1, 5.0, 9.6, 5.5, 5.1, 11.4, 8.0, 12.1, 7.5, 8.5, 13.1, 6.4, 18.0, 27.0, 18.9, 10.8, 13.1, 8.4, 16.9, 2.7, 9.6, 4.5, 12.4, 5.5, 12.7, 17.1. Create a frequency histogram and relative frequency histogram using 5 classes.
    b. The following are yearly income levels for twenty randomly selected Alaska residents for the year 1999 (selected randomly from the 2000 U.S. Census): 9000, 10600, 60000, 80000, 5000, 2600, 14800, 4500, 45000, 23000, 21300, 50000, 17000, 49000, 85000, 38000, 64000, 25000, 23000, 55000. Construct frequency and relative frequency histograms using 6 classes.
7. Pie-charts: observed frequencies and relative frequencies (heading toward chi-square tests in statistics)
    a. Example: The following data give game rating preferences by gender for randomly selected college students. These data were gathered by Sean Meshkin, honors student at Mt. San Antonio College. Create a pie-chart to summarize the proportions represented by these values.

Rated-E	*Rated-T*	*Rated-M*
12	29	20

Chapter Two – Modeling with Linear Functions

Section 2.1: Laboratory Activity – Using Linear Functions to Model Motion

1. Intro and lab overview
2. Definition of linear function – graphically and algebraically
3. Students will learn to use TI-Nspire coupled with CBR motion detector to gather paired sample data for time and position.
4. One student holds the CBR while the other attempts to walk toward it at constant speed. Experiment is repeated until students are able to create a scatter plot that is nearly linear.
5. Students fit linear function to scatter plot, and record coefficients.

Section 2.2: Algebraic Analysis of Linear Equations

1. Intro and section overview
2. Steps to solving linear equations
    a. Examples with whole numbers
        i. $3-2(5x-4)=-25+2x$
        ii. $2(3-2x)-5=3x+15$
    b. Examples with fractions & multiplying by LCM
        i. $\frac{2}{3}x+\frac{3}{4}=\frac{1}{2}x-\frac{7}{8}$
        ii. $x-2\left(\frac{1}{3}x+\frac{2}{5}\right)=\frac{2}{15}-\frac{1}{5}x$
    c. Examples with decimals & multiplying by a power of 10
        i. $2.3\cdot(3-2x)+1.2=-2.2x-2.7$
        ii. $3.2x-2.6=19.4+2.2\cdot(2-4x)$
3. Linear equations in applications and literal equations
    d. fixed cost + variable cost
        i. Suppose a company's fixed costs are 21,500 per month, and produces units at a cost of $150 each. Assuming the company has no other costs and a budget of $100,000 per month, how many units can they produce per month?
    e. Cell phone billing
        i. Suppose your cell phone carrier costs $50 per month, and your plan includes 300 minutes of talk time. After 300 minutes, there is a fee of $0.10 per minute. Suppose your bill for a given month is $63.70. How minutes did you talk on the phone that month?
    f. Account balance with simple interest $A = P + P\cdot r\cdot t$.
        i. Suppose that you invest $1000 (US$) in a Canadian company at 8% simple interest.
        (a) In how many years will the investment grow to $1500 (US$)?
        (b) If the exchange rate is $1.20 Canadian dollars for every US dollar, what is the value of the $1500 in Canadian dollars?
    g. Rectangular region perimeters with known relationship between sides (sports fields)
        i. Suppose that a rectangle has a perimeter of 40 meters, and that the length is 5 meters longer than the width. What are the dimensions of the rectangle? What are the dimensions in *feet*?

Section 2.3: Graphing and Modeling with Linear Functions

1. Intro and section overview
2. Linear functions, and their domain and range
    - *a.* Increasing linear function example
        - *i.* $f(x) = 2x - 3$
    - *b.* Decreasing linear function example
        - *i.* $g(x) = -\frac{1}{2}x + 2$
3. Slope of graph and as rate of change
    - *a.* Definition & formula of slope: $m = \dfrac{y_2 - y_1}{x_2 - x_1}$
    - *b.* Increasing functions and positive slope
        - *i.* $y = 2x - 3$
    - *c.* Decreasing functions and negative slope
        - *i.* $y = -\frac{1}{2}x + 2$
4. Slope-intercept form
    - *a.* Slope-intercept form pure example
    - *b.* Slope-intercept form in application
        - *i.* Fixed cost = *y*-intercept
        - *ii.* Variable cost = slope
5. Point-slope form
    - *a.* Development of point-slope form from the slope formula
        - *i.* In the formula $m = \dfrac{y_2 - y_1}{x_2 - x_1}$, we allow the point (x_2, y_2) to vary, and rename it (*x*, *y*), and multiply both sides by $(x - x_1)$ to obtain $y - y_1 = m \cdot (x - x_1)$.
    - *b.* Point-slope pure example
        - *i.* Find the equation of the line with slope 2/3 and passing through (3, –2).
    - *c.* Point-slope in application example
        - *i.* Suppose a factory uses $150,000 to produce 450 units, and $100,000 to produce 200 units. Find the linear function which computes the total cost to produce *x* units. What are the fixed and variable costs for this factory?
    - *d.* Derivation of slope-intercept form from point-slope form
        - *i.* $y - y_1 = m \cdot (x - x_1)$
$y = m \cdot (x - x_1) + y_1$
$y = m \cdot x + (y_1 - m \cdot x_1)$
$y = m \cdot x + b$
6. Graphing linear functions: point/intercept & slope
    - *a.* Procedure
        - *i.* Pick 3 *x* values randomly
        - *ii.* Compute corresponding *y* values using: $y = m \cdot x + b$
    - *b.* Examples
        - *i.* Graph $y = f(x)$: $f(x) = 2x - 3$
        - *ii.* Graph $y = g(x)$: $g(x) = -\frac{1}{2}x + 2$
        - *iii.* Graph $y = h(x)$: $h(x) = 1.4x + 2.6$
7. Horizontal and vertical lines
    - *a.* Definition & slope
        - *i.* Horizontal line – zero slope

ii. Vertical line – no slope

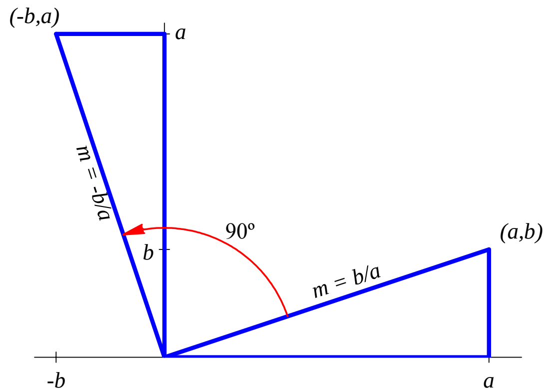
b. Examples

i. Give the equation and graph the line which passes through (2, 5) and (4, 5).

ii. Give the equation and graph the line which passes through (3, 4) and (3, –2).

8. Parallel and perpendicular lines

a. Derivation of perpendicular slopes whose product is –1… proof by picture.



b. Find equations of lines parallel or perpendicular to given lines

i. Find the equation of the line containing the point (2, –3) which is perpendicular to the line $2x + 5y = 3$.

ii. Find the equation of the line containing the point (4, 1) which is perpendicular to the line $-x + 3y = 1$.

c. Determine whether lines are parallel, perpendicular, or neither

i. $2x + 5y = 10$ and $4x + 10y = 4$.

ii. $3x + y = 6$ and $x - 3y = 3$.

iii. $2x + 5y = 4$ and $5x + 2y = 4$.

9. Modeling and problem solving with linear functions

a. *y*-Intercept as initial value, slope as rate of change

i. Suppose an employee of a given company is scheduled to receive a $0.37 per hour raise every month.

1. If she currently earns $12.29 per hour, construct a function which models her pay rate, *r*, in *t* months.
2. Graph this function.
3. When we she earn $17.00 per hour?

b. Relationship between °F and °C.

i. The relationship between degrees Fahrenheit and Celsius is given by the equation, $F = \frac{9}{5}C + 32$.

1. Graph this function on an *CF*-coordinate system.
2. Convert 75º F to Celsius.

c. Simple interest.

i. Account balance with simple interest $A = P + P \cdot r \cdot t$. Suppose $1000 is deposited at 15% simple interest.

1. Find the function that describes the value on the investment at time *t*.
2. Graph this function.
3. When will the investment be worth $1350?

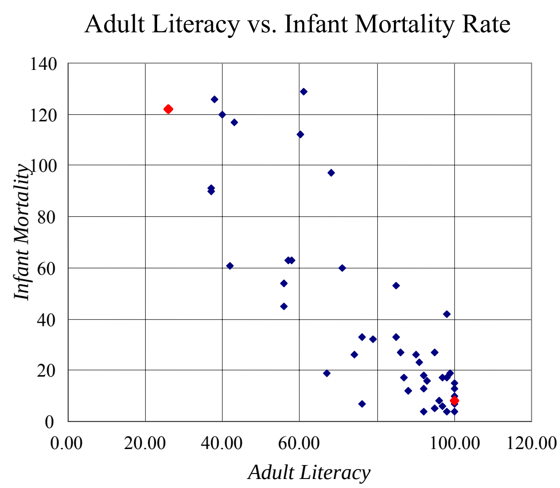
d. Literacy vs. infant mortality

i. Below is a scatter-plot of (*x*, *y*) data points, where each point represents two coordinate values from a given country. The first coordinate, *x*, represents the literacy rate of the given country (the percentage of people over age 17 who can read), while *y* represents the infant mortality rate (deaths of children under 1 year of age per 100,000).

Adult Literacy Rate (2000)	Infant Mortality Rate (2003)
74	26
92	18
85	53
86	27
40	120
43	117
100	11
98	42
68	97
100	15
58	63
42	61
87	17
92	13
26	122
100	4
67	19
98	18
76	7
61	129
71	60
91	23
96	8
97	6
100	13
100	10
98	4
96	8
99	19
37	90
85	33
92	4
37	91
100	7
93	16
98	4
79	32
97	17
76	33
90	26
60	112
98	17
95	27
88	12
56	45

Two representative data points in this collection are (100, 7) and (26, 122).

1. Interpret these points in context.
2. Find the equation of the line which passes through these points. Graph this line.
3. Interpret the slope *y*-intercept of this line
4. For a country with 50% literacy, estimate the infant mortality rate.
5. For a country with 80% infant mortality, estimate the literacy rate.



e. Compute sales from total of sales + tax

i. Derive a function which computes *y*, the total amount paid (price + tax) of an item of price *x* at 8% sales tax.

1. Graph this function
2. Suppose an item costs $74.95. What is the total cost to be paid?
3. Suppose that the total paid for an item is $106.59. What was the item's price?

f. Something about the environment.

10. Reading graphs
    a. Estimate y for a given x
    b. Estimate rate of change as slope
11. Point-slope form and error: $\Delta y = m \cdot \Delta x$, measured error vs. computed error, absolute and relative error.
    a. Absolute/relative error in °F corresponding to absolute/relative error in °C.
        i. Suppose that a temperature is measured in degrees Fahrenheit with a 0.2º maximum error. The temperature is then converted to degrees Celsius. Compute the maximum error in degrees Celcius.
    b. Simple rate problem
        i. Suppose a gas pump delivers a given number of gallons to your gas tank with a maximum error of 0.025 gallons. If the price per gallon is $3.75, what is the maximum error in the amount of money you are charged?
    c. Account balance with simple interest $A = P + P \cdot r \cdot t$.
        i. Suppose you invest $5000 at 8% simple interest for a given amount of time. When your investment value is computed the time, t, is rounded to the nearest month (1/12 year). What is the resulting error in your investment value?

Section 2.4: Absolute Value and Inequalities

1. Absolute Value Properties
    a. Definition of absolute value
    b. Properties of absolute value
    c. Dealing with absolute value equations ($|u| = a \Rightarrow u = \pm a$)
        i. Solve $|x| = 12$
        ii. Solve $|x + 4| = 8$
    d. Properties of Inequalities
    e. Solving Inequalities
        i. Solve and graph: $4x + 3 \leq 9$
        ii. Solve and graph: $2 - 4x > 10$
        iii. Solve and graph: $3 + 2(2 - 4x) \geq 2x + 5$
    f. Dealing with absolute value inequalities ($|u| < a \Rightarrow -a < u < a$). (These are needed for analysis of absolute maximum error. There is no need for the "greater-than" inequalities).
    g. Examples:
        i. Solve and graph: $|x| < 7$
        ii. Solve and graph: $|2x + 3| < 9$

Section 2.5: Analysis of Experimental Data

1. Intro and lab overview
2. Finishing the Lab Report: predicting and interpreting the slope and intercepts using coefficients computed in section 4.1, comparing these predictions to those determined experimentally, identifying sources of error between predicted and observed results.
3. Identifying sources of error.

Chapter Three – Modeling with Linear Systems of Equations

Section 3.1: Laboratory Activity – Systems of Equations and a Mixing Problem

Activity: Mixing Solutions & Hydrogen Ions

1. Intro and lab overview
2. Overview of pH, and conversion to H+ concentration.
3. Statement of the problem: diluting a chemical to create a solution of predetermined volume & pH.
4. Setting up the system of equations.

Section 3.2: Solving Linear Systems

1. Intro and section overview
2. Solution as ordered pair or triple
    a. Example: $\begin{cases} 2x-3y=1 \\ x+2y=4 \end{cases}$, solution $(x, y) = (2, 1)$.
    b. Example: $\begin{cases} x+2y-3z=-4 \\ 2x-y+z=3 \\ 3x+2y-z=4 \end{cases}$, solution $(x, y, z) = (1, 2, 3)$.
3. Solutions by graphing,
    a. Finding intersection of lines
    b. Easy example: $\begin{cases} 2x-3y=1 \\ x+2y=4 \end{cases}$
    c. Hard example
        i. Different lines with no intersection – no solution: $\begin{cases} 2x+4y=2 \\ 3x+6y=2 \end{cases}$
        ii. Different lines with one intersection point – one solution: $\begin{cases} 2x-3y=1 \\ x+2y=4 \end{cases}$
        iii. The same line – infinitely many solutions & their parameterization: $\begin{cases} 2x+4y=2 \\ 3x+6y=3 \end{cases}$
4. Solutions by substitution
    a. Example: $\begin{cases} 2x-3y=1 \\ x+2y=4 \end{cases}$
    b. Example: $\begin{cases} 3x+y=2 \\ 2x-2y=3 \end{cases}$
5. Solutions by elimination
    a. Elementary matrices
    b. Two 2×2 examples and three 3×3 examples
        i. Without fractions
            1. $\begin{cases} 2x-3y=1 \\ x+2y=4 \end{cases}$
            2. $\begin{cases} x+2y-3z=-4 \\ 2x-y+z=3 \\ 3x+2y-z=4 \end{cases}$

ii. With fractions

1. $\begin{cases} \frac{3}{4}x + \frac{5}{3}y = -\frac{1}{15} \\ \frac{5}{6}x - \frac{2}{3}y = \frac{14}{15} \end{cases}$

iii. With decimal fractions

1. $\begin{cases} 1.2x + 2.5y = 3.26 \\ 1.1x - 4.5y = 17.93 \end{cases}$
2. $\begin{cases} 3.4x + 3.7y + 8.4z = 13.34 \\ -2.1x + 2.4y + 3.4z = 12.07 \\ 5.3x - 1.2y - 1.6z = -0.95 \end{cases}$
3. $\begin{cases} 5.4x + 3.2y - 1.0z = 51.0 \\ 2.3x + 4.5y + 6.7z = 29.31 \\ -3.8x + 1.2y + 0.2z = 9.06 \end{cases}$

6. Classification: consistent, inconsistent, infinitely many solutions.
   a. Example of parameterization (one parameter only…) of infinitely many solutions:
   $\begin{cases} 2x + 4y = 2 \\ 3x + 6y = 3 \end{cases}$
   b. No solution example: $\begin{cases} 2x + 4y = 2 \\ 3x + 6y = 2 \end{cases}$
7. Cramer's Rule for 2×2 Linear Systems
8. Applications in Systems of Equations
   a. Combining investments:
      i. Suppose that someone invests 5000 in two accounts with 7% and 12% rates, respectively. If the total interest earned is $528, what amounts were invested in each account?
   b. Mixing:
      i. Suppose two salt solutions are to be mixed to create a 25 liter salt solution with 5 gm of salt per liter. The solutions to be mixed contain 2 gm per liter and 10 gm per liter respectively. How much of each solution should be used?
9. ***Finishing the Lab Report***: Solve the system of equations set-up in section 3.1. Experimentally combine the fluids with quantities determined by the solution to the systems of equations. Test the pH of the solution to determine if observed result matches that which was theoretically predicted.
10. Discuss sources of error.

Section 3.3: The Essentials of Vector Algebra

1. Intro and section overview
2. Position & displacement vectors: algebraic and graphic
   a. Position vector definition
   b. Scalar Definition
   c. Examples: ***a*** = [1, 4] and ***b*** = [3, 1]
3. Displacement vectors and vector addition
   a. Graphic definition
   b. Algebraic definition
   c. Example: position ***a*** = [1, 4] and displacement ***b*** = [3, 1]
4. Scalar multiplication
   a. Graphic definition – positive and negative scalar
   b. Algebraic definition

   c. Example: $\boldsymbol{c} = [3, 2]$, $2\cdot\boldsymbol{c} = 2\cdot[3, 2] = [2\cdot3, 2\cdot2] = [6, 4]$.

5. Additive inverse
   a. Graphically
   b. Algebraically
   c. Example: $\boldsymbol{c} = [3, 2]$, then $-\boldsymbol{c} = -1\cdot\boldsymbol{c} = -\boldsymbol{c} = [-3, -2]$.
6. Vector Addition and Subtraction
   a. Graphically
   b. Algebraically
   c. Example: $\boldsymbol{a} = [1, 4]$ and $\boldsymbol{b} = [3, 1]$
7. Magnitude & slope
   a. Formulas
   b. Examples: $\boldsymbol{a} = [3, 1]$ and $\boldsymbol{b} = [1, 4]$, $|\boldsymbol{a}| = \sqrt{3^2+1^2} = \sqrt{10} \approx 3.162$, $|\boldsymbol{b}| = \sqrt{1^2+4^2} = \sqrt{17} \approx 4.123$. The slope of $\boldsymbol{a}$ is $m_{\boldsymbol{a}} = 1/3$, and the slope of $\boldsymbol{b}$ is $m_{\boldsymbol{b}} = 4/1 = 4$.
   c. Vertical vector – no slope, horizontal vector – zero slope.
      i. Example: $[x, 0]$ has zero slope for $x \neq 0$.
      ii. Example: $[0, y]$ has no slope for $y \neq 0$.
8. Dot product and perpendicular vectors
   a. Proof using slope for 2 dimensional vectors
   b. Three dimensional vectors and associated dot product
   c. Examples. Graph the following vector pairs, then use the dot product to determine whether the following vector pairs are perpendicular or not.
      i. [2, 1], [–3, 6]
      ii. [3, 2], [4, 6]
      iii. [6, 2], [–9, 3]
      iv. [2, 1, 5], [3, 4, –2]
      v. [4, 2, 3], [1, 3, –3]

Section 3.4: Matrices and Row-Echelon Form

1. Intro and section overview
2. Definition of matrix and augmented matrix
    a. Example: $\begin{cases} x+2y-3z=-4 \\ 2x-y+z=3 \\ 3x+2y-z=4 \end{cases}$, $\mathbf{A}=\begin{bmatrix} 1 & 2 & -3 \\ 2 & -1 & 1 \\ 3 & 2 & -1 \end{bmatrix}$, $\boldsymbol{b}=\begin{bmatrix} -4 \\ 3 \\ 4 \end{bmatrix}$, $\mathbf{A}\# = [\mathbf{A}|\boldsymbol{b}] = \left[\begin{array}{ccc|c} 1 & 2 & -3 & -4 \\ 2 & -1 & 1 & 3 \\ 3 & 2 & -1 & 4 \end{array}\right]$
3. Elementary operations and upper triangular form
    a. Add any multiple of a first row to a second row, replacing the second row by this sum.
    b. Reorder the rows of the matrix.
    c. Multiply any row by a non-zero scalar.
4. Matrix elimination procedure
    a. 2×2 example and 3×3 examples
        i. With whole number solutions: $\begin{cases} 3x-y+z=8 \\ 2x+2y-3z=7 \\ 4x-3y-2z=4 \end{cases}$, $\mathbf{A}\# = \left[\begin{array}{ccc|c} 3 & -1 & 1 & 8 \\ 2 & 2 & -3 & 7 \\ 4 & -3 & -2 & 4 \end{array}\right]$
        ii. With fractions
            1. $\begin{cases} \frac{3}{4}x+\frac{5}{3}y=-\frac{1}{15} \\ \frac{5}{6}x-\frac{2}{3}y=\frac{14}{15} \end{cases}$, $\mathbf{A}\# = \left[\begin{array}{cc|c} \frac{3}{4} & \frac{5}{3} & -\frac{1}{15} \\ \frac{5}{6} & -\frac{2}{3} & \frac{14}{15} \end{array}\right]$
        i. With decimal fractions
            1. $\begin{cases} 1.2x+2.5y=3.26 \\ 1.1x-4.5y=17.93 \end{cases}$, $\mathbf{A}\# = \left[\begin{array}{cc|c} 1.2 & 2.5 & 3.26 \\ 1.1 & -4.5 & 17.93 \end{array}\right]$
            2. $\begin{cases} 3.4x+3.7y+8.4z=13.34 \\ -2.1x+2.4y+3.4z=12.07 \\ 5.3x-1.2y-1.6z=-0.95 \end{cases}$, $\mathbf{A}\# = \left[\begin{array}{ccc|c} 3.4 & 3.7 & 8.4 & 13.34 \\ -2.1 & 2.4 & 3.4 & 12.07 \\ 5.3 & -1.2 & -1.6 & -0.95 \end{array}\right]$
            3. $\begin{cases} 5.4x+3.2y-1.0z=51.0 \\ 2.3x+4.5y+6.7z=29.31 \\ -3.8x+1.2y+0.2z=9.06 \end{cases}$, $\mathbf{A}\# = \left[\begin{array}{ccc|c} 3.4 & 3.7 & 8.4 & 13.34 \\ -2.1 & 2.4 & 3.4 & 12.07 \\ 5.3 & -1.2 & -1.6 & -0.95 \end{array}\right]$
5. Infinitely many, no solutions
    a. Infinitely many: $\begin{cases} 2x+4y=2 \\ 3x+6y=3 \end{cases}$, $\mathbf{A}\# = \left[\begin{array}{cc|c} 2 & 4 & 2 \\ 3 & 6 & 3 \end{array}\right]$
    b. No solution: $\begin{cases} 2x+4y=2 \\ 3x+6y=2 \end{cases}$, $\mathbf{A}\# = \left[\begin{array}{cc|c} 2 & 4 & 2 \\ 3 & 6 & 2 \end{array}\right]$

Section 3.5: Operations on Matrices

1. Intro and section overview
2. Matrix transpose
   a. Definition

   b. Example: For the matrix $\mathbf{A} = \begin{bmatrix} 2 & -1 & 5 \\ 3 & 2 & 1 \end{bmatrix}$, the transpose is $\mathbf{A}^{\mathrm{T}} = \begin{bmatrix} 2 & 3 \\ -1 & 2 \\ 5 & 1 \end{bmatrix}$.
3. Matrix multiplication
   a. Dot product as row vector times column vector $\boldsymbol{a}\cdot\boldsymbol{b} = \boldsymbol{a}^{\mathrm{T}}\boldsymbol{b}$

   i. For the column vectors $\boldsymbol{a} = \begin{bmatrix} 3 \\ 2 \\ 7 \end{bmatrix}$ and $\boldsymbol{b} = \begin{bmatrix} 1 \\ 6 \\ -4 \end{bmatrix}$, the dot product is written as $\boldsymbol{a}\cdot\boldsymbol{b} =$

   $\boldsymbol{a}^{\mathrm{T}}\cdot\boldsymbol{b} = [3, 2, 7]\cdot\begin{bmatrix} 1 \\ 6 \\ -4 \end{bmatrix} = 3\cdot1+2\cdot6+7\cdot(-4) = 3 + 12 - 28 = -13.$

   b. Definition of matrix multiplication using dot products

   c. Example: $\mathbf{A} = \begin{bmatrix} 2 & -3 & 1 & 3 \\ 1 & -2 & 4 & 3 \end{bmatrix}_{2x4}$ and $\mathbf{B} = \begin{bmatrix} 1 & 0 & -1 \\ 3 & 2 & -2 \\ -1 & 1 & 2 \\ 3 & -1 & 2 \end{bmatrix}_{4x3}$, $\mathbf{A}_{2x4}\cdot\mathbf{B}_{4x3} =$

   $$\begin{bmatrix} 2 & -3 & 1 & 3 \\ 1 & -2 & 4 & 3 \end{bmatrix}\cdot\begin{bmatrix} 1 & 0 & -1 \\ 3 & 2 & -2 \\ -1 & 1 & 2 \\ 3 & -1 & 2 \end{bmatrix}$$
4. Matrix form of a linear system
   a. Example: Suppose $\mathbf{A} = \begin{bmatrix} 2 & 4 & 3 \\ 3 & 5 & 6 \\ 6 & 2 & 9 \end{bmatrix}$, and $\boldsymbol{x} = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$. Note that $\mathbf{A}\cdot\boldsymbol{x} = \begin{bmatrix} 2 & 4 & 3 \\ 3 & 5 & 6 \\ 6 & 2 & 9 \end{bmatrix}\cdot\begin{bmatrix} x \\ y \\ z \end{bmatrix} =$

   $\begin{bmatrix} 2x+4y+3z \\ 3x+5y+6z \\ 6x+2y+9z \end{bmatrix}$. Next, suppose that $\boldsymbol{b} = \begin{bmatrix} 2 \\ 1 \\ 4 \end{bmatrix}$, and consider the equation $\mathbf{A}\cdot\boldsymbol{x} = \boldsymbol{b}$. This evaluates as: $\mathbf{A}\cdot\boldsymbol{x} = \begin{bmatrix} 2 & 4 & 3 \\ 3 & 5 & 6 \\ 6 & 2 & 9 \end{bmatrix}\cdot\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 2x+4y+3z \\ 3x+5y+6z \\ 6x+2y+9z \end{bmatrix} = \begin{bmatrix} 2 \\ 1 \\ 4 \end{bmatrix} = \boldsymbol{b}$. From this we see that $2x+4y+3z=2$, $3x+5y+6z=1$, and $6x+2y+9z=4$. Thus the equation $\mathbf{A}\cdot\boldsymbol{x} = \boldsymbol{b}$ is equivalent to the system $\begin{cases} 2x+4y+3z=2 \\ 3x+5y+6z=1 \\ 6x+2y+9z=4 \end{cases}$. Because of this $\mathbf{A}\cdot\boldsymbol{x} = \boldsymbol{b}$ is called the *vector form* of this system of linear equations.

Section 3.6: Least-Squares Solutions

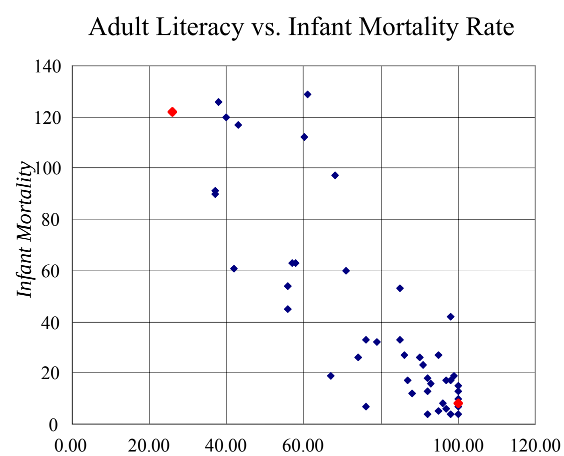
1. Intro and section overview
2. Motivation using a $2x1$ over-determined system
   a. Example: $\begin{cases} 2x = 3 \\ 3x = 4 \end{cases}$
3. Generalization to an *mxn* over-determined system
4. Example done step by step – no technology: $\begin{cases} 2x - y = 0 \\ x + 2y = 5 \\ x + y = 4 \end{cases}$
5. Application: linear regression
   a. Linear regression example by hand – three data points
      i. Draw a scatter-plot for the data points: (0, 2), (1, 0), and (2, -1).
      ii. Find the linear function, $y = mx + b$, which most nearly passes through these points. Do this by hand and show all work. Add the graph of this model to your scatter-plot from above.
      iii. Use the linear model above to estimate a value for y corresponding to $x = 3$.

b. Linear regression using technology: TI calculator/Excel/Web application.

c. The table below gives infant mortality rates corresponding to literacy rates for a random sample of
countries.

Adult Literacy Rate (2000)	Infant Mortality Rate (2003)
74	26
92	18
85	53
86	27
40	120
43	117
100	11
98	42
68	97
100	15
58	63
42	61
87	17
92	13
26	122
100	4
67	19
98	18
76	7
61	129
71	60
91	23
96	8
97	6
100	13
100	10
98	4
96	8
99	19
37	90
85	33
92	4
37	91
100	7
93	16
98	4
79	32
97	17
76	33
90	26
60	112
98	17
95	27
88	12
56	45

1. Use technology to generate a scatter-plot for these points.
2. Find the equation of the regression line for these points. Graph this line on the scatter-plot from above.
3. Interpret the slope and *y*-intercept of this line
4. For a country with 50% literacy, use the regression line to estimate the infant mortality rate.
5. For a country with 80% infant mortality, use the regression line to estimate the literacy rate.



d. The meaning of R^2.

Chapter Four – Modeling with Quadratic Functions

Section 4.1: Laboratory Activity – Using Quadratics to Model Projectile Motion

Activity: Bouncing Ball Experiment
Key elements: Students use bouncing ball & TI-CBR bouncing ball program to:

1. Intro and lab overview
2. Capture bouncing ball data;
3. Select a single parabolic subset of the captured data;
4. Trace the graph to find intercepts and vertex;
5. Least-squares fit of data points to a parabola, noting *a*, *b*, and *c* coefficients.
6. Note possible sources of error.
7. Lab report begins by summarizing the findings above.

Section 4.2: Algebraic Analysis of Quadratic Equations

1. Intro and section overview
2. Definitions – quadratic equation, coefficients, 2^{nd} degree/2 solutions
3. Approximating Zeros Using the Intermediate Value Theorem
    a. Looking for zeros using sign changes
        i. Example: $8x^2 = 30x + 27$
        ii. Counterexample: $25x^2 + 9 = 30x$
4. The Square Root Property
    a. Two examples of solving a quadratic by extracting roots.
        i. $x^2 = 9$
        ii. $(2x+7)^2 = 25$
    b. Imaginary solutions $\sqrt{-a} = \sqrt{a}i$
        i. $(3x-5)^2 = -16$
        ii. $-2(x-7)^2 = 30$
5. Completing the Square to Solve Quadratic Equations
    a. Leading coefficient equal to one
        i. Positive *x*-coefficient
            1. $x^2 + x = 20$
        ii. Negative *x*-coefficient
            1. $x^2 + 7 = 6x$
        iii. Fractional terms added
            1. $x^2 + 5x + 3 = 0$
        iv. Imaginary
            1. $x^2 + 13 = 4x$
    b. Leading coefficient not equal to one
        i. Positive *x*-coefficient
            1. $2x^2 + 8x = 5$
        ii. Negative *x*-coefficient
            1. $-3x^2 + 18x = -4$
        iii. Fractional terms added
            1. $3x^2 + 7x = 6$
            2. $2x^2 = 5x + 12$
6. Finding Exact Zeros Using the Quadratic Formula

a. Developing the quadratic formula

b. Applying the formula

  i. Without double negative inside discriminant

    1. $x^2 + 7 = 6x$

  ii. With double negative

    1. $3x^2 + 7x = 6$

  iii. Imaginary Solution

    1. $x^2 + 13 = 4x$

7. Solving literal quadratic equations

  a. Areas of spheres

    i. The area of a sphere of radius r is given by: $A = 4\pi \cdot r^2$. Solve for *r*.

  b. Projectile motion – solve for time

    i. When an object are vertically thrown upward or downward, its height, *h*, at time *t* (in seconds) is given by the equation $h = -\frac{1}{2}gt^2 + v_0 t + h_0$. In this equation, upward is considered *positive* for the variable *h*, *g* is – 9.9 m/s^2 in metric or –32 ft/sec^2 in English units, v_0 represents initial velocity (which is negative if the object is thrown downward), and h_0 represents initial height. Suppose an object is projected upward from a height of 20.3 meters and an initial upward velocity of 15 meters per second.

      1. When will the object obtain a height of 27 meters?
      2. When will the object land on the ground?

  c. Rectangular regions with unknown sides but know relationship between them

    i. Suppose a rectangular field is 51 feet longer than it is wide, and that its total area is 4,462 feet. What are the dimensions of the field?

  d. Inverse square with known intensity but unknown distance

    i. The intensity of radiation from the Sun is 9140 watts per square meter at the distance of Mercury (0.387AU); but only 1370 watts per square meter at the distance of Earth (1AU)—a threefold increase in distance results in a ninefold decrease in intensity of radiation. This tells us that light intensity obeys the inverse square law – that light intensity varies inversely with the square of the distance to the light source.

      1. What is the light intensity at Mars, whose distance to the Sun is 1.1.542 AU? (The average distance from Earth to the Sun is 1 AU – Astronomical Unit, and the conversion factor is 1 AU = 149597870.74 meters).
      2. At what distance from the Sun is light intensity equal to 2000 watts per square meter? How many miles is this? How many kilometers is this?

8. Characterizing Solutions using the Discriminant

  a. Two real solutions: $6x^2 = 5x + 6$

  b. One real solution: $9x^2 + 16 = 24x$

  c. Two imaginary solutions: $9x^2 + 13 = 12x$

Section 4.3: Graphing and Modeling with Quadratic Functions

1. Intro and section overview
2. Quadratic functions and their domain & range
    a. Definition of quadratic function & its domain
        i. Example: $f(x)=x^2+6x-3$
    b. Completing the square on a quadratic function: $y = a(x-h)^2 + k$, $h = -b/(2\cdot a)$
        i. Example: $f(x)=x^2+6x-3$
        ii. Example: $g(x)=-2x^2+3x+9$
    c. Range of a quadratic function: $y = a(x-h)^2 + k$, range: $y \geq k$ ($a > 0$) or $y \leq k$ ($a < 0$).
        i. Optimization of quadratic function: $y = k$. Min if $a > 0$, max if $a < 0$.
            1. Example: $h(x)=2(x-3)^2+1$
            2. Example: $f(x)=x^2+6x-3$
            3. Example: $g(x)=-2x^2+3x+9$
3. Graphing Quadratic Functions: intercepts, vertex (focus?), range, axis of symmetry, long-range behavior.
    a. Example: $k > 0$, $a > 0$ (no intercepts)
        i. Graph $y = f(x)$: $f(x)=x^2-4x+13$
    b. Example: $k = 0$ (one intercept)
        i. Graph $y = g(x)$: $g(x)=4x^2-4x+1$
    c. Example: $k > 0$, $a < 0$ (two intercepts)
        i. Graph $y = h(x)$: $h(x)=2x^2+8x-5$
    d. Example: $k < 0$, $a < 0$ (no intercepts)
        i. Graph $y = f(x)$: $f(x)=-x^2+2x-4$
    e. Example: $k < 0$, $a > 0$ (two intercepts)
        i. Graph $y = g(x)$: $g(x)=2x^2-4x$
4. Optimization and Quadratic Modeling
    a. Projectile motion
        i. When an object are vertically thrown upward or downward, its height, h, at time t (in seconds) is given by the equation $h=-\frac{1}{2}gt^2+v_0t+h_0$. In this equation, upward is considered *positive* for the variable h, g is – 9.9 m/s^2 in metric or –32 ft/sec^2 in English units, v_0 represents initial velocity (which is negative if the object is thrown downward), and h_0 represents initial height. Suppose an object is projected upward from a height of 20.3 meters and an initial upward velocity of 15 meters per second.
            1. At what time does the object reach its maximum height (at the parabola's vertex)?
            2. What is the maximum height in *meters* and also *feet*?
    b. Rectangular region with constraint
        i. Suppose you have 100 feet of fence material, and you would like to fence a rectangular region on three sides, with the fourth side bordered by a building.
            1. Construct a quadratic function which represents the area of the rectangular region as a function of the width, w, of the region.
            2. Use the quadratic function constructed above to determine the dimensions of the rectangular region with maximum area.
            3. What is the area of the maximized region in ft^2 and m^2.
5. Reading graphs

Section 4.4: Fitting Paired Data to Quadratic Functions

1. Intro and section overview
2. Easy example of a least squares fit of a quadratic to 4 data points – by hand. This would include analysis and interpretation of intercepts and vertex.
   a. Draw a scatter plot of the data pairs below.
   b. Fit a quadratic model $y = ax^2 + bx + c$ to these points. Sketch the graph of this quadratic onto the scatter-plot from above. Do this by hand, and show all work.
   c. Use the model determined above to estimate *y* corresponding to *x* = 2.

x	*y*
–2	–1
–1	1
0	2
1	0

3. Larger example using software. Software: Minitab, Excel, or other online tool (programmed in Java by Math Lab Tech). This also would include analysis and interpretation of intercepts and vertex.
   a. The table below contains current in milliamps for 16 batteries made of identical materials except for varying electrolyte solutions. It is known that the current produced by a battery is somewhat dependent on the pH of the electrolyte.
      i. Sketch a scatter-plot of the paired data.
      ii. Use technology to fit a quadratic function to the paired data below. Add the graph of this function to your scatter-plot from above.
      iii. Give and interpret the R^2 value.
      iv. Estimate the current produced by this battery with an electrolyte of 4.000 pH.

pH	Current
2.604	3.2
3.999	1.6
5.029	1.2
6.152	1.0
7.664	1.0
2.613	3.1
3.668	1.6
4.936	1.5
5.963	1.3
7.182	1.3
10.055	1.3
8.320	0.9
9.510	1.2
11.460	5.8
10.682	2.5
9.545	1.9
8.427	1.4
7.664	0.9

4. Finishing the Lab Report: predicting the vertex and intercepts using coefficients computed in section 4.1, comparing these predictions to those determined experimentally, identifying sources of error between predicted and observed results.

Chapter Five – Modeling with Rational Functions

Section 5.1: Laboratory Activity – Using Rational Functions to Discover Boyle's Law

Activity: Discovering Boyle's Law (or Inverse Square Law in Sound)

1. Intro and lab overview
2. For each sample member, estimate volume and use the pressure probe to compute pressure. Record volumes with sampled pressure values.
3. Graph a scatter plot.
4. Least-squares fit of sample data to various models, use R^2 to pick the best model. State Boyle's law for this particular example.
5. Estimate pressures for given volumes based on model. Compare observed pressure values to estimated values.
6. Note possible sources of error. Lab report begins by summarizing the findings above.

Section 5.2: Algebraic Analysis of Rational Functions

1. Intro and section overview
2. Domain and range for rational functions in inverse proportions
    - *a.* $f(x) = \dfrac{3}{x^2}$
    - *b.* $g(x) = \dfrac{4x}{2x+1}$
3. Reducing rational expressions
    - *a.* No factoring required in examples
        - *i.* $\dfrac{2}{8x}$
        - *ii.* $\dfrac{8a^2}{12a^5}$
4. Adding and subtracting rational expressions
    - *a.* No factoring required in examples
        - *i.* $\dfrac{2x}{4} + \dfrac{3x}{6}$
        - *ii.* $\dfrac{5}{2y} - \dfrac{7}{6y}$
        - *iii.* $\dfrac{1}{R_1} + \dfrac{1}{R_2}$
        - *iv.* $2y - \dfrac{3}{y}$
5. Multiplying and dividing rational expressions.
    - *a.* No factoring required in examples
        - *i.* $\dfrac{2}{5x} \cdot \dfrac{15x^2}{8}$
        - *ii.* $\dfrac{3}{5y} \cdot \dfrac{4}{12y}$
        - *iii.* $\dfrac{2a}{15} \div \dfrac{8a^2}{10}$

iv. $2x \div \frac{8x}{9}$

v. $\frac{15z}{8} \div 3z^3$

6. Simplifying simple complex fractions by inverting and multiplying.
   a. No factoring required in examples
      i. $\dfrac{\frac{3x}{5}}{\frac{9x}{15}}$
      ii. $\dfrac{1}{\frac{1}{R_1} + \frac{1}{R_1}}$
      iii. $\dfrac{\frac{2}{x}+3}{\frac{4}{x}+1}$
      iv. $\dfrac{\frac{2}{y}+\frac{5}{2y}}{\frac{1}{y}+\frac{3}{2y}}$
7. Solving proportions and rational literal equations, particularly of the form $a/x + b = c$.
   a. Literal Equations and Inverse Variation:
      i. For the equation $y = k/x$, solve for k if $x = 4.7$ and $y = 8.2$.
      ii. Suppose that x varies with the inverse of y squared. When $x = 7$, $y = 25$. What y value corresponds to $x = 10$?
      iii. When two resistors are connected in series in an electrical circuit, the resistances combine according to the principle: $\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2}$.
         1. Solve for R
         2. Solve for R_1.
      iv. The Ideal Gas Law is represented by the equation: $\frac{PV}{T} = nR$. Solve for T.
   b. Abstract Equations
      i. $\frac{4}{x^2} = 25$
      ii. $\frac{25.34}{4.7} = \frac{15.3}{2.5x}$
      iii. $\frac{7}{x} - \frac{3}{4x} = \frac{3}{4}$
      iv. $\frac{2}{4x} - \frac{1}{2} = \frac{1}{8}$
      v. $\frac{3}{8} + \frac{5}{12x} = \frac{2}{3}$
      vi. $3 + \frac{4}{x} - \frac{5}{x^2} = 1$

vii. $2.6 - \frac{1.2}{x} = \frac{2.4}{x^2}$

c. Applications

i. If one pump can fill a swimming pool in 15 hours, and a second can fill the same pool in 9 hours, how long would be required to fill the pool with both pumps working together?

ii. Suppose a given can painter can paint 1000 square feet in 6 hours. Suppose also that when the painter is joined by a second painter, the same work is done in 2 hours. In how many hours can the second painter paint 800 square feet?

c. Upstream/Downstream problems

i. Suppose a stream flows at 2 km/hr. A boat can move 5 km upstream in the same time that it can move 10 km downstream. How fast does the boat move in still water?

ii. What is this speed in miles per hour?

8. Review of unit conversions

Section 5.3: Graphing and Modeling with Rational Functions

1. Intro and section overview
2. Graphs of Rational Functions, $y = k/x^n$, for various values of n.
    a. Domain and range
        i. $n = 1$
        ii. $n = 2$, $k > 0$ or $k < 0$.
    b. Asymptotic behavior – asymptotes are the coordinate axes.
    c. Examples
        i. Graph $y = f(x)$: $f(x) = \frac{1}{x}$
        ii. Graph $y = g(x)$: $g(x) = \frac{-2.8}{x}$
        iii. Graph $y = h(x)$: $h(x) = \frac{1}{x^2}$
        iv. Graph $y = f(x)$: $f(x) = \frac{-3.5}{x^2}$
3. Deterministic models with rational functions. Asymptotes in applications.
    a. Rate = Distance/Time
        i. For a fixed distance, explain the behavior of rate of travel as time approaches zero.
        ii. For a fixed distance, explain the behavior of rate as time becomes larger and larger.
    b. Inverse square Gravity: $F=k/r^2$. where r is the distance to a black hole – an object with mass compressed to a point in space.
        i. Explain the behavior of force due to gravity as we approach a black hole.
        ii. Explain the behavior of force due to gravity as we move further and further from the black hole.
4. Reading graphs & interpreting points and asymptotes

Section 5.4: Fitting Paired Data to Rational Functions

1. Intro and section overview
2. Easy example of a least squares fit of a rational function to 4 data points – by hand. This would include analysis and interpretation of asymptotes.
    a. Draw a scatter-plot for the points (2.4, 1.1), (4.3, 0.6), (0.7, 3.6).
    b. Fit the model $y = k/x$ to the data points above. Graph this model on the scatter-plot from above.
    c. Using the model derived above, predict a y value corresponding to $x = 1.5$.
3. Larger example using software. Software: Minitab, Excel, or other online tool (programmed in Java by Math Lab Tech). This also would include analysis and interpretation of asymptotes.
    a. The data below were gathered for a fixed quantity of gas particles, under various pressures (in *kPa*) and corresponding volumes (in cm^3) recorded below.
        i. Sketch a scatter-plot of the data points in a *PV*-coordinate system.
        ii. Since Boyle's law states that these measurements vary inversely to one-another, fit the data points below to the model $P = k/V$. Add the graph of this curve to the scatter-plot from above.
        iii. Estimate the volume of this gas corresponding to a pressure of 300 *kPa*.
        iv. Explain the behavior of the pressure as the gas is compressed to near zero volume.
        v. Explain the behavior of the pressure as gas is allowed to expand over a larger and larger space.

Pressure (*kPa*)	Volume (cm^3)
100	41
350	12
50	79
200	22
400	9
250	18
25	160

4. Finishing the Lab Report: Identifying sources of error, conjecture regarding relationship between pressure and volume, leading to Boyle's Law.

Chapter Six – Modeling with Radical Functions

Section 6.1: Laboratory Activity – Using Radical Functions to Model Light Intensity

Activity: Light Intensity. In this activity, students will study graphs of light intensity at various distances from a light sensor. They will use power and linear regressions to determine the relationship between light intensity and distance from the light source. They will also develop their own models for the relationship.

1. Intro and lab overview
2. A sequence of regularly spaced intervals of length, sample a light intensity from a flashlight. Record distances with sampled light intensity measurements. In this case, we will use light intensity as the independent variable, and distance as the dependent variable.
3. Graph a scatter plot.
4. Least-squares fit of power function to data points. Estimate distances for given light intensities. $I = k/d^2$ implies $d = \sqrt{k/I}$.
5. Compare observed distances to predicted values.
6. Note possible sources of error.

Section 6.2: Algebraic Analysis of Radical Functions

1. Intro and section overview
2. Domain and range for radical functions: square and cube roots only.
    - *a.* Example: Find domain (ray pointing right)
        - *i.* Find the domain of the function $f(x) = \sqrt{3x+6}$
    - *b.* Example: Find domain (ray pointing left)
        - *i.* Find the domain of the function $g(x) = \sqrt{12-8x}$
    - *c.* Example: Find domain (all reals)
        - *i.* Find the domain of the function $h(x) = \dfrac{4x}{\sqrt{x^2+9}}$
    - *d.* Example: Find the domain (division and radical)
        - *i.* Find the domain of the function $k(x) = \dfrac{2x-5}{\sqrt{3x+12}}$
    - *e.* Example with cube roots:
        - *i.* Find the domain of the function $f(x) = \sqrt[3]{2-9x}$
3. Simplifying/adding/subtracting/multiplying/dividing radical functions.
    - *a.* Review of properties of radicals
        - *i.* Evaluate expressions with radicals on calculator
            1. Evaluate: $2.7/\sqrt{105.4}$
            2. Evaluate: $\dfrac{12-10.4}{7.2/\sqrt{11.8}}$
            3. Evaluate: $\dfrac{25-21.3}{\sqrt{\dfrac{11.5^2}{13}+\dfrac{7.3^2}{21}}}$
    - *b.* Add/subtract example (like radicals initially). No calculators allowed on these.
        - *i.* Collect terms: $5\sqrt{10} - 13\sqrt{10}$
        - *ii.* Collect terms: $7 \cdot \sqrt[3]{6x} + 6 \cdot \sqrt[3]{6x}$
    - *c.* Add/subtract example (like radicals after simplifying). No calculators allowed on these.

i. Collect terms: $3\sqrt{80}-2\sqrt{20}$

ii. Collect terms: $2\sqrt{5x}+7\sqrt{45x}$

iii. Collect terms: $11\sqrt{8x^3}+4\sqrt{18x^3}$

d. Multiply example (no simplification needed). No calculators allowed on these.

i. $\sqrt{10}\sqrt{3}$

ii. $3\sqrt{7x}\sqrt{5}$

iii. $4\cdot\sqrt[3]{4x}\cdot\sqrt[3]{5}$

iv. $8\cdot\sqrt[3]{-5}\cdot\sqrt[3]{25}$

e. Multiply example (simplification needed after multiplying). No calculators allowed on these.

i. $4\sqrt{5}\sqrt{10}$

ii. $2\sqrt{8x}\sqrt{20}$

f. Dividing radical expressions & rationalizing denominators – review

i. Example – one term with radical in denominator. No calculators allowed on these.

1. Simplify: $\dfrac{4}{\sqrt{5}}$
2. Simplify: $\dfrac{-3}{2\sqrt{7}}$

ii. Example – one term with radical in denominator plus radical in numerator. No calculators allowed on these.

1. Simplify: $-\sqrt{\dfrac{3}{5}}$
2. Simplify: $\dfrac{5\sqrt{2}}{4\sqrt{3}}$

4. Solving radical equations and literal equations, particularly of the form $a\cdot\sqrt[n]{x}+b=c$, or $a/\sqrt[n]{x}+b=c$. No calculators allowed on these.

a. Literal Equation Examples:

i. Solve for *I*: $d=\sqrt{k/I}$.

ii. Solve for *x*: $t=\sqrt{2x/g}$.

b. Abstract Equation Examples

i. $25=2\sqrt{2x}+17$

ii. $-\dfrac{80}{\sqrt{4x}}+10=2$

iii. $\dfrac{2}{\sqrt{3x}}+\dfrac{4}{\sqrt{12x}}=\dfrac{4}{3}$

5. Extracting cube roots

a. The volume of a sphere of radius *r* is given by $A=\frac{4}{3}\pi\cdot r^3$

b. A box with a square base and height which is twice as tall as the length of the base has volume equal to 54 ft^3.

i. What are the dimensions of the box?

ii. What is the volume of the box in meters cubed?

Section 6.3: Graphing and Modeling with Radical Functions

1. Intro and section overview
2. Graphs of radical functions, $y = k\cdot\sqrt[n]{x}$, for various values of *n*. Long range behavior.
   a. Example: $n = 2$, $k > 0$
      i. Graph $y = f(x)$: $f(x) = \sqrt{x}$
   b. Example: $n = 2$, $k < 0$
      i. Graph $y = g(x)$: $g(x) = -1.8\sqrt{x}$
   c. Example: $n = 3$, $k > 0$
      i. Graph $y = h(x)$: $h(x) = \sqrt[3]{x}$
   d. Example: $n = 3$, $k < 0$
      i. Graph $y = f(x)$: $f(x) = -2.5\sqrt[3]{x}$
3. Deterministic models with rational functions.
   a. Example: Something with an inverse square law. Sound maybe?
   b. Example: Something with gravity? $x = ½\cdot g\cdot t^2$, $t = \sqrt{2x/g}$.
      i. The time, *t* (in seconds), for an object to fall *x* feet vertically is given by the formula $t = \sqrt{2x/g}$, $g = –9.8$ m/sec. How many seconds would be required for an object to fall 100 feet? Note – gravity is given in meters per second, and a unit conversion will be required.
4. Reading graphs.

Section 6.4: Fitting Paired Data to Radical Functions

1. Intro and section overview
2. Easy example of a least squares fit of a radical function to 4 data points – by hand. This would include analysis and interpretation of asymptotes.
3. Larger example using software. Software: Minitab, Excel, or other online tool (programmed in Java by Math Lab Tech). This also would include analysis and interpretation of asymptotes.
4. Finishing the Lab Report: Identifying sources of error, conjecture regarding relationship between light intensity and distance.

Chapter Seven – Modeling With Exponential and Logarithmic Functions

Section 7.1: Laboratory Activity – Using Exponential Functions to Model Cooling

Activity: Newton's Law of Cooling

1. Intro and lab overview
2. Students monitor temperature of warm water in degrees Celsius after ice is added.
3. Students fit an exponential function to data points, noting equation provided by calculator.

Section 7.2: Algebraic Analysis of Exponential Functions

1. Intro and section overview
2. Definition and purpose of the exponential function
   - *a.* Example of exponential function – doubling function $f(x) = 2^x$
   - *b.* The Natural Exponential: $g(x) = e^x$
   - *c.* Another example: $h(t) = -25.5 \cdot e^{-5.1t}$
3. Domain and range for exponential functions.
   - *a.* Example with (0,∞) range
     - *i.* Give the domain and range for the function $f(x) = 2^x$.
     - *ii.* Give the domain and range for the function $g(x) = e^x$
   - *b.* Example with (–∞,0) range
     - *i.* Give the domain and range for the function $h(t) = -25.5 \cdot e^{-5.1t}$
4. Evaluating exponential expressions with a calculator: y^x, e^x, and 10^x buttons.
   - *a.* Round to four places after the decimal:
     - *i.* $2 \cdot e^{4.7}$
     - *ii.* $153.7 \cdot 10^{-2.5}$
     - *iii.* $65 \cdot 47.9^{-0.75}$
5. Graphs of exponential functions.
   - *a.* Graph the function $y = f(x)$: $f(x) = 2^x$
   - *b.* Graph the function $y = g(x)$: $g(x) = e^x$
   - *c.* Graph the function $y = h(x)$: $h(t) = -25.5 \cdot e^{-5.1t}$
6. Modeling with exponential functions and doubling time
   - *a.* Compound interest growth
     - *i.* $A(t) = P\left(1 + \frac{r}{n}\right)^{nt}$
   - *b.* Time value of money
     - *i.* $A(t) = \left(\frac{m \cdot n}{r} + P\right)\left(1 + \frac{r}{n}\right)^{nt} - \frac{m \cdot n}{r}$
   - *c.* Population Growth/Drug elimination/Radioactive decay
     - *i.* Growth: $A(t) = A_0 e^{kt}$, $k > 0$.
     - *ii.* Decay: $A(t) = A_0 e^{-kt}$, $k < 0$.

Section 7.3: Inverse Functions

1. Intro and section overview
    a. Inverse function & solving equations process
        i. Example: Solve the equation $y = f(x)$ for x, where $f(x) = 2x - 1$
        ii. Make note: Opposite operations in opposite order
2. Definition of the inverse function
    a. Give $f^{-1}(x)$ where $f(x) = 2x - 1$ and show that $f(f^{-1}(x)) = f^{-1}(f(x)) = x$
3. Determining the inverse of a function algebraically
    a. Procedure – input variable & output variable switch
    b. Easy Example
        i. Example: Let $g(x) = \frac{3}{x} - 4$. Find $g^{-1}(x)$ and verify that $g(g^{-1}(x)) = g^{-1}(g(x)) = x$.
    c. Example with restricted domain
        i. Let $f(x) = \sqrt{x-2}$. Find $f^{-1}(x)$.
        ii. Note that the domain of f^{-1} is the range of f, domain of f is the range of f^{-1}
4. The graph of the inverse function
    a. Relationship between function and inverse function graphs.
    b. Easy one to one example
        i. Graph $y = f(x)$ and its inverse on the same axes, where $f(x) = 2x - 1$.
    c. Exponential function inverse
        i. Graph $y = g(x)$ and its inverse on the same axes, where $g(x) = 2^x$.
    d. Example where inverse is not a function
        i. Graph $y = h(x)$ and its inverse on the same axes, where $h(x) = 2x^2 - 8$.
5. One to one functions and the horizontal line test and existence of the inverse function
    a. Repeat above example in context of the horizontal line test
        i. Does $y = h(x)$ satisfy the horizontal line test, where $h(x) = 2x^2 - 8$?
    b. Does $y = f(x)$ satisfy the horizontal line test, where $f(x) = 2x - 1$?

Section 7.4: Logarithms and Their Properties

1. Intro and section overview
2. Definition of the logarithm in the context of inverse functions
   a. For the function, $g(x)=2^x$, what is the inverse function?
   b. $g^{-1}(x)=$ the power to which 2 must be raised to yield $x=\log_2(x)$.
3. Domain and range of logarithmic functions
   a. Domain = { x; $x > 0$ }
      i. Give the domain for $f(x)=\log_3(2x)$
      ii. Give the domain for $g(x)=\log_2(x-3)$
   b. Range = $\mathbb{R}$
      i. Give the range for $f(x)=\log_3(2x)$
4. Evaluating logarithms
   a. To evaluate $\log_b(x)$, simply ask "*b* to the *what* yields *x*?"
   b. $\log_3(9)$
   c. $\log_{10}(1000)$
   d. $\log_4\left(\frac{1}{64}\right)$
   e. $\log_7(1)$
   f. $\log_5(-5)$
5. Special Cases
   a. Natural Logarithm
      i. Evaluate:
         1. $\ln\left(e^{4.5}\right)$
         2. $\ln\left(e^{3x}\right)$
         3. Using calculator
            a. ln(14.27)
            b. ln(3.72)
   b. Common Logarithm
      i. Evaluate:
         1. $\log(10{,}000)$
         2. $\log(0.000001)$
         3. $\log\left(10^{4.3}\right)$
         4. Using calculator
            a. log(36.71)
            b. log(0.0041)
            c. log(–121.6)
6. Properties of logarithms
   a. Switching logarithmic/exponential forms
      i. $y=\log_b(x) \Leftrightarrow b^y = x$
      ii. Examples: Solve by switching forms
         1. Solve by switching from exponential to logarithmic form (round to four places after the decimal):
            a. $10^x = 245.6$
            b. $10^{2x} = 0.0042$
            c. $e^x = 25.7$

d. $e^{-3x} = 0.0201$

b. Logarithm of exponential expression – independent bases: $\log_b(x^n) = n \cdot \log_b(x)$

$$y = n \cdot \log_b(x)$$
$$y / n = \log_b(x)$$
$$b^{y/n} = x$$
$$b^y = x^n$$
$$y = \log_b(x^n)$$
$$\log_b(x^n) = n \cdot \log_b(x)$$

c. Solving exponential equations
  i. Apply natural logarithm to each side to solve
    1. $2^x = 25.5$
    2. $3^{-4x} = 0.0062$

d. Change of base
  i. Development
  ii. Examples
    1. Evaluate to four places after the decimal
      a. $\log_2(25)$
      b. $\log_7(213.9)$
      c. $\log_{0.5}(15.3)$

7. Solving logarithmic equations
  a. Most basic forms – solved by changing to exponential form
    i. Solve $\log_2(x) = 5.5$
    ii. Solve $\ln(5x) = 1.7$
    iii. Solve $2.7 \cdot \log(3x) = 5.513$
    iv. Solve $-4.3 \cdot \log_{3.1}(3x) = 28.9$
8. Graphing logarithmic functions
  a. Graph by considering the inverse function's graph.
    i. Graph $y = \log_2(x)$ and $y = 2^x$ together.
  b. Most basic forms – graph by switching to exponential form
    i. Graph $y = f(x)$, where $f(x) = \log_{2.2}(x)$.
    ii. Graph $y = g(x)$, where $g(x) = 4\log_{0.5}(2x)$
9. Logarithmic Scales
  a. pH example
    i. Suppose a solution has a pH of approximately 1.05. What is the H+ ion concentration?
    ii. Suppose a solution has a H+ ion concentration of 0.015 moles per liter. What is the approximate pH of the solution
  b. Decibel example
  c. Richter example

Section 7.5: Modeling with Exponential Functions

1. Intro and section overview
2. Solving exponential literal equations using logarithms.
3. Deterministic models with exponential functions.
    a. Asymptotes and intercepts in applications.
    b. Using logarithms to solve for independent variable in the exponent.
        iii. Compound interest growth: $A(t) = P\left(1 + \frac{r}{n}\right)^{nt}$
        iv. Time value of money: $A(t) = \left(\frac{m \cdot n}{r} + P\right)\left(1 + \frac{r}{n}\right)^{nt} - \frac{m \cdot n}{r}$
        v. Population Growth/Drug elimination/Radioactive decay
            1. Growth: $A(t) = A_0 e^{kt}$, $k > 0$.
            2. Decay: $A(t) = A_0 e^{-kt}$, $k < 0$.
                a. Newton's law of cooling – use degrees Celsius for a model that approaches zero, then convert to Fahrenheit.
                b. Charge on electric capacitor
4. Reading graphs.

Section 7.6: Fitting Paired Data to Exponential Functions

1. Intro and section overview
2. Easy example of a least squares fit of an exponential function to 4 data points – by hand. This would include analysis and interpretation of asymptotes and intercepts.
    a. Use least squares methods to fit the points below to the model $y = b \cdot e^{mx}$.

x	y
1.5	10.1
2.9	37.1
3.3	53.8
4.7	197.8

3. Larger example using software. Software: Minitab, Excel, or other online tool (programmed in Java by Math Lab Tech). This also would include analysis and interpretation of asymptotes and intercepts.
    a. During the 1980s the population of a certain city went from 100,000 to 205,000. Populations by year are listed in the table below.

Year	1990	1991	1992	1993	1994	1995	1996	1997	1998	1999
Population	100,000	108,000	117,000	127,000	138,000	149,000	162,000	175,000	190,000	205,000

    Use computer software to find an exponential model that mostly approximates this growth trend. Based on this model, predict a time when the town's population will reach 250,000.

4. Finishing the Lab Report: Finding the inverse of the fitted exponential function to predict times for given temperatures using logarithms, interpreting asymptotes and intercepts, identifying sources of error.

Chapter Eight – Summary of Critical Mathematical Methods

Section 8.1: Review of Ratios, Proportions, and Literal Equations

1. Intro and section overview
2. Ratios and Unit Multipliers
3. Solving proportions with unit conversions in applications
    a. If 12 gallons of gas costs $42.48, how much would 37 liters cost?
    b. If a car driving at constant speed travels 182.7 miles in 2.9 hours, how many kilometers can it travel in 4.1 hours?
    c. If a 174 pound patient requires 435 mg of a given drug every day, how much would a 200 pound patient require?
    d. If 8 gallons of paint are required to paint 245 square feet, how many gallons would be required to paint 720 square feet?
4. Solving proportions using percentages in applications
    a. If an alcohol and water solution is to be mixed with 36% alcohol, and we want to use 8 gallons of alcohol, how many liters of water should be added?
    b. If 16% of a bag of Skittles is colored red, with 12 red skittles total, how many Skittles are in the bag?
5. Isolating variables by addition, subtraction, multiplication, and division rules.
    a. Solve for *h*: $A = \frac{1}{2}(b + B)h$
    b. Solve for *b*: $A = \frac{1}{2}(b + B)h$
    c. Solve for *r*: $V = \frac{4}{3}\pi \cdot r^3$
    d. Solve for *t*: $x = \frac{1}{2}at^2$
    e. Solve for r: $A = P + P \cdot r \cdot t$

Chapter Nine – Sequence, Series, and Probability

Section 9.1: Laboratory Experiment – Bouncing Ball Again

Activity: Bouncing ball – measuring bounce ratios

1. Use TI CBR to conduct the bouncing ball experiment again
2. Record the total time for bounce sequence and initial height
3. Compute ratio of max height of given bounce to max height of next bounce
4. Do for several bounce sequence pairs & compute the average ratio

Section 9.2: Introduction to Sequences and Series

1. Intro and section overview
2. Notation for sequence and series
3. Evaluating Series
    - *a.* $\sum_{n=1}^{5} n^2$
    - *b.* $\sum_{n=1}^{4} n!$
4. Arithmetic Series
    - *a.* Definition: difference between terms is constant
    - *b.* Examples:
        - *i.* $\sum_{n=3}^{9} (n-3)$
        - *ii.* $\sum_{n=1}^{5} (2n-1)$
    - *c.* Formula for evaluating arithmetic series
        - *i.* $\sum_{n=1}^{100} n$
5. Evaluating Geometric Series – Finite and Infinite
    - *a.* Finite number of terms
        - *i.* $\sum_{n=1}^{5} 3 \cdot 2^n$
        - *ii.* $\sum_{n=1}^{4} \left(\frac{1}{2}\right)^n$
    - *b.* Formula for finite geometric series
        - *i.* Evaluate $\sum_{n=1}^{\infty} \frac{3}{2^n}$
        - *ii.* Evaluate $\sum_{n=1}^{\infty} 0.1^n$
6. Using geometric Series to model real-world phenomena
    - *a.* If an employee is hired making $30,000 per year, with a 6% pay increase every year for the next 30 years, how much money is earned during that time?
    - *b.* Suppose you invest $5000 per year in an annuity which earns 10% interest, compounded annually. How much will the annuity be worth in 30 years?
7. Lab Analysis:
    - *a.* Use Geometric Series to estimate total bounce time and distance.

Section 9.3: The Binomial Theorem and Binomial Experiments

1. Intro and section overview
2. Factorials and Combinations
    a. The Purpose and Definition of Factorials
    b. Evaluating Factorials
        i. 5!
        ii. 20!
        iii. 0!
    c. The Purpose and Definition of Combinations
        i. ${}_5C_3$
        ii. ${}_{10}C_6$
        iii. ${}_7C_7$
    d. Pascal's Triangle
3. The Binomial Theorem
    a. Expand $(x+y)^5$
    b. Expand $(2x+3)^6$
4. Basics of Probability – Meaning without Computation
5. Discrete random variable
6. Discrete probability distribution
    a. Defined & Properties
    b. Determine if a collection of discrete probabilities represents a discrete distribution of probabilities.
        i. A deck of cards has 52 cards with 13 hearts. A gambling game consists of a player drawing three cards without replacement. The required bet is $1000. The winnings depend on the number of hearts drawn out of the three cards. The net winnings and their probabilities are below, with one probability missing.

Outcome	*x* = Net Winnings	*P(x)*
0 hearts	–1000	0.413
1 heart	100	0.436
2 hearts	750	**?**
3 hearts	20000	0.013

What is the missing probability?

7. The binomial distribution
    a. Defined with formula
    b. Example:
        i. Suppose that 59% of all people would like to see gun control laws strengthened. In sampling 10 people randomly (with replacement), let *x* represent the number of people who would like to see gun control laws strengthened. What is the probability that exactly 6 of the 10 people surveyed would like gun control strengthened?
8. Proof that the sum of binomial probabilities equals one using the Binomial Theorem.