Introduction to Radicals

Radicals will be: evaluated, simplified, added, subtracted, multiplied and divided.

- Square root or radical notation \( \Rightarrow \sqrt[n]{\text{Radicand}} \)

What number multiplied by itself gives the number under the radical?

- Perfect square numbers evaluate to rational numbers
  - \( \sqrt{25} = 5 \) Because \( 5 \cdot 5 = 25 \)

- Other numbers require the use of a calculator and evaluate to irrational number
  - \( \sqrt{13} = 3.605551275... \)

- Can you take the square root of a negative number? Why?
  - \( \sqrt{-16} = ??? \) Does \( (-4)(-4) = -16 \) No. So, the answer is not a real number.

Square roots have an index of 2 other radicals are possible index 3, 4, 5 and so on. But the definition is the same.

- \( \sqrt[3]{125} = 5 \) \textbf{Because,} \( 5 \cdot 5 \cdot 5 = 125 \) or \( \sqrt[4]{81} = 3 \) \textbf{Because,} \( 3 \cdot 3 \cdot 3 \cdot 3 = 81 \) and so on.

Simplifying radicals

What is the square root of a variable raised to a power? \( \sqrt{x^6} = x^3 \) Why? Because \( x^3 \cdot x^3 = x^6 \)

Another way to think about it is divide the power by the index. \( \sqrt{x^6} = x^{6/2} = x^3 \) If the power does not divide evenly then the variable must be factored first so that the power can be divided by the index.

Example: \( \sqrt[4]{x^{14}} = x^{14/4} \) The power does not reduce to a whole number. So, factor the part that does:

\[ \sqrt[4]{x^{12}x^2} \Rightarrow \sqrt[4]{x^4} \sqrt[4]{x^2} \Rightarrow x^3 \sqrt[4]{x^2} \]

\( \Rightarrow \) The remainder stays inside the radical.

Next Example: Simplify

- \( \sqrt{108x^5y^8} \) Problem? 108 is not a perfect square but one of its factors maybe.

- \( \sqrt{(36)(3)x^5y^8} \) What about \( x^5 \) 5 divide 2 does not work, so that must be factored as well.

- \( \sqrt{(36)(3)x^4y^8} \) Everything that can be simplified comes out of the radical and everything else must stay inside.

\[ \sqrt{(36)(3)x^4y^8} \Rightarrow 6x^2y^4\sqrt{3x} \Rightarrow 6x^2y^4\sqrt{3x} \]

The same rules apply to all other radicals just the index changes.

Example:

\[ \frac{3}{5}\sqrt[3]{135x^7y^{15}} \Rightarrow \frac{3}{5}\sqrt[3]{(27)(5)x^6y^{15}} \Rightarrow \frac{3}{5}x^2y^5\sqrt[5]{5x} \]

Answer: \( 3x^2y^5 \sqrt[5]{5x} \) Reduce all terms.
Multiply/Divide radicals: If the indexes are the same write as one radical.

Multiply

- $\sqrt[4]{2x^3y} \cdot \sqrt[4]{8x^5y^2}$: The indexes are the same "4". So, write as one radical first.
- $\sqrt[4]{2x^3y} \cdot \sqrt[4]{8x^5y^2}$: Everything inside will multiply. Note: Remember your power rules.
- $\sqrt[4]{2 \cdot 8x^3y^4 y^2} \Rightarrow \sqrt[4]{16x^8 y^6} \Rightarrow \text{Reduce} \Rightarrow 2x^2 \sqrt[4]{y^3}$

Example: $2 \sqrt[3]{27x}$ is read as $2 \times \sqrt[3]{27x}$ When the radical is reduced whatever comes out of the radical is multiplied by what is already out front.

- $2 \sqrt[3]{27x} \Rightarrow 2 \cdot 3 \sqrt[3]{x} \Rightarrow 6 \sqrt[3]{x}$ Done!

Example: Multiply

- $5x \sqrt[4]{4x^3} \cdot 6x^2 \sqrt[4]{18x^4}$: Since this is multiplication we can reshuffle all terms.
- $5x \cdot 6x^2 \sqrt[4]{4x^3} \cdot \sqrt[4]{18x^4} \Rightarrow 30x^3 \sqrt[4]{72x^7} \Rightarrow \text{Reduce the radical}$
- $30x^3 \sqrt[4]{36 \cdot 2x^6 x} \Rightarrow 30x^3 \cdot 6x^3 \sqrt[4]{2x} \Rightarrow 180x^6 \sqrt[4]{2x}$ Done!

Divide

- $\sqrt[3]{\frac{162x^7 y^{10}}{6x^4 y}}$: The indexes are the same “3”. So, write as one big radical first.
- $\sqrt[3]{\frac{162x^7 y^{10}}{6x^4 y}} \Rightarrow \sqrt[3]{27x^3 y^9} \Rightarrow \text{Simplify} \Rightarrow 3xy^3$

Adding/Subtracting radicals: The same as like terms. If all the terms match then combine the number out front.

Example: $8\sqrt{11x} + 5\sqrt{11x} - 7\sqrt{11x} \Rightarrow (8 + 5 - 7)\sqrt{11x} \Rightarrow 6\sqrt{11x}$

Some radicals may need to be reduced first before they are added.

Example:

- $5x^2 \sqrt{3x} + 4 \sqrt{12x^5} - 2x \sqrt{27x^3}$: Term do not match
- $5x^2 \sqrt{3x} + 4 \sqrt{4 \cdot 3x^4} - 2x \sqrt{9 \cdot 3x^2 x}$: Reduce each radical if possible
- $5x^2 \sqrt{3x} + 4 \cdot 2x^2 \sqrt{3x} - 2 \cdot 3x \cdot x \sqrt{3x}$: Multiply terms out front
- $5x^2 \sqrt{3x} + 8x^2 \sqrt{3x} - 6x^2 \sqrt{3x}$: Add like radicals $7x^2 \sqrt{3x}$ Done!
Practice Problems
Perform the indicated operation and reduce all radicals.

1) $\sqrt{128x^3}$

2) $\sqrt{243m^5n^2}$

3) $\sqrt{27a^2}$

4) $\sqrt{2} \cdot \sqrt{8}$

5) $-7 \sqrt[3]{3y^2} \cdot \sqrt[3]{18y}$

6) $\sqrt[3]{25p} \cdot \sqrt[3]{125p^2}$

7) $\sqrt[4]{\frac{4x^2}{25y^4}}$

8) $\sqrt[3]{48x^3y^7} \div \sqrt[3]{3xy}\frac{1}{3}$

9) $\sqrt[4]{8m^7} \cdot \sqrt[4]{6m^3}$

10) $5\sqrt{7} - 4\sqrt{7} + 2\sqrt{7}$

11) $3x\sqrt{7} + \sqrt[3]{28x^2} - \sqrt[3]{63x^2}$

12) $\sqrt[3]{50} + \sqrt[3]{98} - \sqrt[3]{75} + \sqrt[3]{27}$

Answer Key

1) $8x\sqrt{2x}$

2) $9m^2n\sqrt{3m}$

3) $3a\sqrt{3}$

4) 4

5) $-21y\sqrt[3]{2}$

6) $5p\sqrt[3]{25p}$

7) $\frac{2x}{5y^2}$

8) $4xy^2$

9) $2m^2\sqrt[4]{3m^2}$

10) $3\sqrt{7}$

11) $2x\sqrt{7}$

12) $12\sqrt{2} - 2\sqrt{3}$