Logarithms and Exponentials

A *logarithmic* function is the inverse of an *exponential* function, and an *exponential* function is the inverse of a *logarithmic* function. For example,

| $f(x) = 2^{x}$ (exponential) | inverse ——— | • $f^{-1}(x) = \log_2 x$ (logarithm) |
|---------------------------------|-------------|---|
| $f(x) = log_2 x$ (logarithm) | inverse ——— | • $f^{-1}(x) = 2^x$ (exponential) |

Note: An exponential function is one with a variable in the exponent. So, $f(x) = b^x$ (where b is a positive constant, $b \neq 1$) is the exponential function, base b. The function $f(x) = x^2$ is NOT an exponential function.

How to *read* logarithms ("logs"):

When you have ... $\log_5 x$... it is read, "the log, base 5, of x." Or when you have ... $\log_2 x^4$... it is read, "the log, base 2, of x to the power of 4."

Note: In the expression $\log_b c$, **b** is the "base" and **c** is called the "argument." (b > 0, c > 0)

Converting LOGARITHMS into EXPONENTIALS and EXPONENTIALS into LOGARITHMS:

"a is equal to the log, base b, of c"

"b to the power of a is equal to c"

 $a = log_b c$ \leftarrow converts to \longrightarrow $b^a = c$ (logarithmic) $b^a = c$

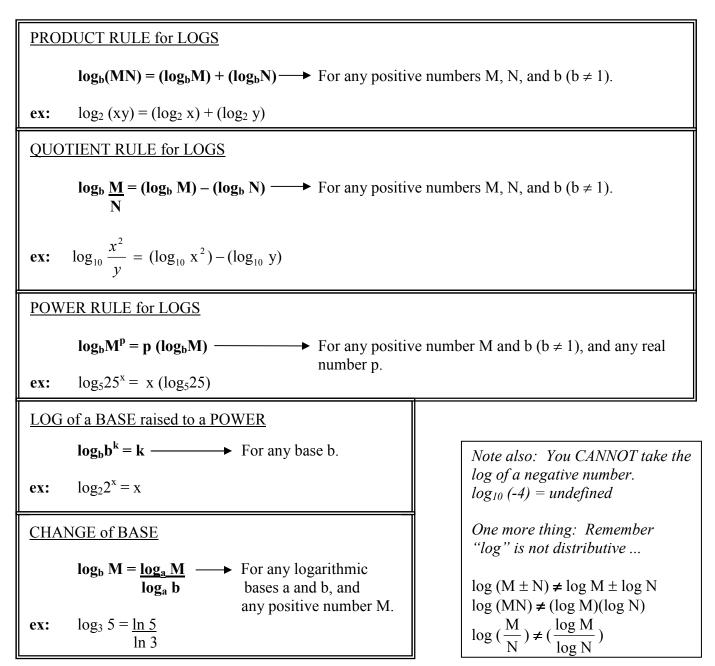
Note: In the logarithmic equation, the base is **b**. This corresponds to the base **b** in the exponential equation. In the exponential equation, the exponent is **a**, and this corresponds to what the entire log is equal to in the logarithmic equation. The logarithm is an exponent. If we can remember how these "positions" of **a**, **b**, **c** convert from logarithms to exponentials and back again, then rewriting logs as exponentials or exponentials as logs won't be a problem.

ex: $9 = 3^x$ converts to $\longrightarrow x = \log_3 9$ $y = \log_3 27$ converts to $\longrightarrow 3^y = 27$

LOGARITHMIC and EXPONENTIAL Equality:

| If then If then | $x = y$ $\log_a x =$ | | | |
|--------------------------|----------------------|--|--|---|
| ex : $4^{(x+1)}$ | $^{)} = 4^{(2x)}$ | Since the bases are both "4," we may equate exponents. | $\mathbf{ex}: \log_2(\mathbf{x}+2) = \log_2 4$ | Same base means we may equate arguments. |
| | | • | x + 2 = 4 | Subtract "2" from both sides. |
| x + 1 | = 2x | Subtract "x" from both | | |
| | | sides. | x = 2 | Answer. The argument must be greater than zero, so $(x + 2) > 0 \dots (2 + 2) > 0$ |
| 1 = x | | Answer. | | $\dots 4 > 0 \dots$ answer checks. |

Logarithmic Properties: The following is a list of some logarithmic properties that may be useful.



Note: When using the change of base formula, we will often change to base 10 (so in the change of base formula, a = 10). In this way, you can use the log button on your calculator to solve the problem. A log with base 10 is called the "common log" and is written without the ten, so $log_{10} 5$ is written log 5. The "natural log" is another log that comes up often. The natural log is just a log with base "e" ("e" is a constant equal to approximately 2.718281828). Natural logs use ln_{10} instead of $log_{e_{10}}$, so $log_{e_{10}} 4$ is written ln 4. The In button is also on most calculators, so you could change to base "e" if you choose.

| ex: $\log x = 3.8$ | Rewrite the common log as a power of ten. | ex: $\ln x = 2$ | Rewrite the natural log as a power of e. |
|--------------------|--|-----------------|---|
| $10^{3.8} = x$ | Exact answer. | $e^2 = x$ | Exact answer. |
| 6309.57 ≈ x | Approximate answer. Checks since $6309.57 > 0$ | 7.39 ≈ x | Approximate answer. Checks since $7.39 > 0$ |