# Dividing Polynomials

The process of dividing polynomials is very similar to the process we use to do long division of rational numbers. If the divisor is a monomial (one term), the process is simplified significantly. If the divisor contains more than one term, the process can be time consuming, but it is not too difficult if we remember some key steps.

Dividing a polynomial by a monomial. If the divisor is a monomial, we will divide each term in the dividend by that monomial term and then simplify.

**ex**: $(40x^3 - 4x^2 + 2x - 7) \div (20x)$

$$\frac{40x^3}{20x} - \frac{4x^2}{20x} + \frac{2x}{20x} - \frac{7}{20x} \longrightarrow$$

Since we are dividing by a monomial, we have divided each term in the dividend by the monomial, 20x.

$$2x^2 - \frac{x}{5} + \frac{1}{10} - \frac{7}{20x} \longrightarrow$$

Simplifying each of these fractions gives us the quotient (the answer).

Dividing a polynomial by a divisor with more than one term. Just like long division of rational numbers, the process will involve dividing, multiplying and subtracting. We will work through an example and explain each step.

**ex**: $(6x^5 - 7x^4 + 4x^3 + 2x - 14) \div (2x^3 - x^2 + 2)$

$$2x^3 - x^2 + 0x + 2\overline{\big)\,6x^5 - 7x^4 + 4x^3 + 0x^2 + 2x - 14}$$

We use the same long division brackets that we used in arithmetic. Notice that the dividend and divisor are in descending order and that we used placeholders like $0x^2$ wherever a term was missing.

$$\begin{array}{rrrrrrr}
 & & & 3x^2 & -\,2x & +\,1 & \\
\cline{2-7}
2x^3 - x^2 + 0x + 2\,\big) & 6x^5 & -\,7x^4 & +\,4x^3 & +\,0x^2 & +\,2x & -\,14 \\
 & \ominus & \oplus & & \ominus & & \\
 & 6x^5 & -3x^4 & +\,0x^3 & +6x^2 & & \\
\cline{2-5}
 & & -4x^4 & +\,4x^3 & -\,6x^2 & +\,2x & \\
 & & \oplus & \ominus & & \oplus & \\
 & & -4x^4 & +2x^3 & +\,0x^2 & -4x & \\
\cline{3-6}
 & & & 2x^3 & -\,6x^2 & +\,6x & -\,14 \\
 & & & \ominus & \oplus & & \ominus \\
 & & & 2x^3 & -\,x^2 & +\,0x & +\,2 \\
\cline{4-7}
 & & & & -5x^2 & +\,6x & -16
\end{array}$$

$$\mathbf{3x^2 = \frac{6x^5}{2x^3}} \qquad \mathbf{-2x = \frac{-4x^4}{2x^3}} \qquad \mathbf{1 = \frac{2x^3}{2x^3}}$$

*You may prefer to create fractions to find the terms of the quotient. The numerator is the first term in the newest sum under the dividend and the denominator is the first term of the divisor. Simplify this fraction to get the next term of the quotient.*

- Divide $2x^3$ into $6x^5$. This quotient, $3x^2$ is written on top of our bracket.
- Multiply $3x^2$ by $(2x^3 - x^2 + 0x + 2)$ to get $(6x^5 - 3x^4 + 0x^3 + 6x^2)$.
- Subtract this expression (we will change the signs and add). To keep the signs correct, we have changed them and circled the new signs (we didn't change the signs for our placeholders since plus and minus zero give the same result). Now we add to get $(-4x^4 + 4x^3 - 6x^2)$.
- We brought down the 2x term just as we would bring down the next digit in arithmetic. Now we repeat the process with the new line of terms.

$$3x^2 - 2x + 1 + \frac{-5x^2 + 6x - 16}{2x^3 - x^2 + 2}$$

Answer. We are finished when the remainder (in this case: $-5x^2 + 6x - 16$) is a lower degree than the divisor.
We write the remainder as a fraction (remainder / divisor).

 

Synthetic Division: Synthetic division is a special way to divide polynomials. It is a great deal faster than long division, but it can only be used with certain divisors. You can only use synthetic division with divisors of the form x – k (where k is a constant). We will work through an example.

**ex**: $(x^4 + 3x^2 + 6x - 10) \div (x + 2)$

Notice that the divisor is of the form x – k. This means that we can use synthetic division. When doing synthetic division, we are concerned with the coefficients in the dividend and k in the divisor of the form x – k. Take note of the minus sign. If the divisor is x – 5, then k = 5, but if the divisor is x + 7, then k = –7 (since x + 7 = x – (–7)).

$$\begin{array}{r|rrrrr} -2 & 1 & 0 & 3 & 6 & -10 \\ & & & & & \\ \hline & & & & & \end{array}$$

First, we set up the problem like this, with k out in front of our inverted division bar and the coefficients of the dividend on the inside. Notice the zero we use as a placeholder since the dividend is missing an $x^3$ term.

$$\begin{array}{r|rrrrr} -2 & 1 & 0 & 3 & 6 & -10 \\ & \downarrow & -2 & & & \\ \hline & 1 & -2 & & & \end{array}$$

multiply

Next, we bring down the first coefficient, in this case the 1. Then we multiply that number by k (which is –2 in this problem). This product (–2) is written below the next coefficient (the zero in this case). Then we add the zero and the –2 and write this sum below the division bar.

$$\begin{array}{r|rrrrr} -2 & 1 & 0 & 3 & 6 & -10 \\ & & -2 & 4 & & \\ \hline & 1 & -2 & 7 & & \end{array}$$

multiply

Now we have a new number below the division bar. We will repeat the process with this new value. So, we multiply (–2) and (–2) and write the product (4) below the next coefficient (3). We add the 3 to the 4, and write the sum (7) below the division bar. Do you recognize the pattern?

$$\begin{array}{r|rrrrr} -2 & 1 & 0 & 3 & 6 & -10 \\ & & -2 & 4 & -14 & 16 \\ \hline & 1 & -2 & 7 & -8 & 6 \end{array}$$

Remainder

The zero-degree coefficient. This will be $-8x^0 = -8$

The first-degree coefficient. This will be $7x^1 = 7x$

The second-degree coefficient. This will be $-2x^2$

The third-degree coefficient. This will be $x^3$

This is the result of our synthetic division. The last number (farthest to the right) under the division bar is the remainder (6). To write our quotient, we look at the numbers that are below the division bar. These numbers are the coefficients for our quotient. The number immediately to the left of the remainder is the zero-degree coefficient (–8); then going from right to left, we increase the degree of our terms. We should end up with a quotient that is one degree less than the degree of our dividend.

$$x^3 - 2x^2 + 7x - 8 + \frac{6}{x+2}$$

Answer. Remember that when doing division, we can write our remainder as a fraction with the remainder in the numerator and the divisor in the denominator.